

Welcome to AP Calculus!

Successful Calculus students must have a strong foundation in algebra and trigonometry. The following packet was designed to help you review your algebra skills in preparation for Calculus. Please do these problems thoughtfully (on separate paper) and check your answers (they are at the end of this document). If you find you need more practice on particular topics, find extra practice online. The stronger your algebra skills are when you enter Calculus, the better off you will be throughout the year.

Because you are planning to take AP Calculus BC, there are some topics you need to learn on your own before starting the course. They are listed at the end of the problems section of this document.

Section 1: Algebra Review

1. Solve for y: $xy - 3x - 2 = 3y$

2. Solve for y: $\ln(y - 1) - \ln x = x + \ln x$

3. Factor completely: $8x^3 + 27$

4. Factor completely: $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

5. Factor completely: $x^2(x - 3) - 4(x - 3)$

6. Simplify $\frac{5(x+h)^2 - 5x^2}{h}$

7. Simplify $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

8. $\frac{3x+2}{(x+3)(x^2-4)} = 0$

Simplify the following. Leave answers in fraction form.

Example:

$$\begin{aligned} \frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)} &= \frac{(x+1)^2[(x+1)(4x-9) - (16x+9)]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[4x^2 - 5x - 9 - 16x - 9]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[4x^2 - 21x - 18]}{(x-6)(x+1)} \\ &= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)} \\ &= (x+1)(4x+3) \end{aligned}$$

$$9. (x-1)^3(2x-3) - (2x+12)(x-1)^2$$

$$10. \frac{(x-1)^2(3x-1) - 2(x-1)}{(x-1)^4}$$

Simplify by rationalizing the numerator:

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2}$$

$$11. \frac{3 - \sqrt{x+9}}{x}$$

$$12. \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

Solve each over the real numbers. Write your answer in interval notation.

$$13. |x-3| \leq 2$$

$$14. |2x+1| > 4$$

$$15. x^2 - 3x - 10 < 0$$

Section 2 : Exponentials and Logarithms

Solve each equation over the real numbers. Round all decimal answers to the nearest thousandth.

$$16. 4^x = 8^{x+1}$$

$$17. 2 \times 3^x + 5 = 17$$

$$18. 5e^{2x} + 4 = 27$$

$$19. \log_3 x = 2$$

$$20. \ln x = 4$$

$$21. \log_3 6 + 2\log_3 x = \log_3 54$$

$$22. \text{Expand using the laws of logs: } \log \frac{3x^2y^3}{z^4\sqrt{w}}$$

$$23. \text{Write as 1 log statement: } 2\ln\sqrt{x} - \frac{1}{3}\ln y + \ln z - 4\ln w$$

Simplify using the fact that $a^{\log_a x} = x$

Example: Simplify $5^{\log_5 25} = 25$

Example: Simplify $e^{\frac{1}{2} \ln 3} = e^{\ln 3^{\frac{1}{2}}} = 3^{\frac{1}{2}}$

Example: $e^{(2 \ln \frac{1}{4})^t} = \left(e^{2 \ln \frac{1}{4}} \right)^t$ using the property of exponents: $(a^b)^c = a^{bc}$ in reverse

$$= \left(e^{\ln \left(\frac{1}{4} \right)^2} \right)^t \quad \text{using the property of logs } a \log b = \log b^a$$

$$= \left(\left(\frac{1}{4} \right)^2 \right)^t \quad \text{using the fact that } a^{\log_a x} = x$$

$$= \left(\frac{1}{4} \right)^{2t} \quad \text{using the property of exponents: } (a^b)^c = a^{bc}$$

24. $e^{t \ln 5}$

25. $e^{\left(\frac{1}{10} \ln \frac{3}{4} \right)^t}$

Section 3 : Trig review

In Calculus, trigonometry shows up randomly so there are things you need to remember. You need to know the 3 Pythagorean Identities, your double angle and power reducing identities and You MUST know your special values.

26. List the Pythagorean identities : 1. 2. 3.

27. Double angle identities: $\cos 2x =$ or or
 $\sin 2x =$

28. Power reducing identities: $\sin^2 x =$

$$\cos^2 x =$$

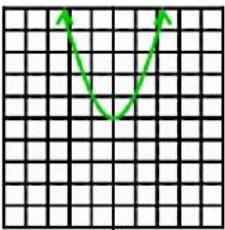
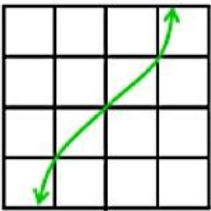
29. Leave all answers in radical form:

a. $\cos\left(\frac{3\pi}{4}\right)$ b. $\sin\left(\frac{7\pi}{6}\right)$ c. $\sec\left(\frac{\pi}{6}\right)$ d. $\sin\left(\frac{\pi}{2}\right)$

e. $\cos(\pi)$ f. $\cot\left(\frac{7\pi}{4}\right)$ g. $\cos\left(\frac{5\pi}{3}\right)$ h. $\sin\left(\frac{\pi}{3}\right)$

i. $\cos\left(\frac{5\pi}{6}\right)$ j. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ k. $\cos^{-1}\left(-\frac{1}{2}\right)$ l. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Section 4: Graphing review.

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

30. Determine algebraically whether each of the following is even or odd.

a. $f(x) = \frac{4x}{x^2 - 3}$

b. $f(x) = \frac{4x}{x - 3}$

c. $f(x) = \frac{4x^2}{x^4 - 3}$

31. You should know the graphs of the basic functions. Graph the following:

a. $f(x) = \sqrt{x}$

b. $f(x) = x^2$

c. $f(x) = x^3$

d. $f(x) = \sin x$

e. $f(x) = \cos x$

f. $f(x) = \tan x$

g. $f(x) = \tan^{-1} x$

h. $f(x) = e^x$

i. $f(x) = \ln x$

Section 5: linear functions

32. Write the equation of the line through (3, 5) with slope $-\frac{2}{3}$. Write your answer in point-slope form.

33. Write the equation of the line through (2, -3) and (-4, 1) in point-slope form.

34. The slope of a line is $-\frac{1}{2}$ and the line passes through the points (2, 5) and (-4, y). Find y.

Section 6: Average Rate of Change

35. Find the average speed of a car that has traveled 350 miles in 7 hours.

36. Suppose $f(1) = 2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

Section 7: Parametric Equations

Sketch the graph, indicating orientation.

37. $x = 2t - 5$, $y = 4t + 3$, $-1 \leq t \leq 4$

38. $x = 4\sin t$, $y = 2\cos t$, $0 \leq t \leq 2\pi$

Section 8: Inverse Functions.

39. Algebraically find the inverse of $y = \frac{2}{x+1} - 5$

40. If (a, b) lies on the graph of $y = f(x)$, what points lie on the graph of $y = f^{-1}(x)$

41. Given the function $f(x) = x^3 + 2x + 1$, find $f^{-1}(4)$.

Section 9: Graphing Calculator Skills

You should be able to do the following using the appropriate functions on your graphing calculator.

- Find an appropriate viewing rectangle for any function so that you can see a complete graph.
- Find x-intercepts (zeros) of a function. These are points.
- Find the intersection of 2 functions. These are points.
- Find the relative maximum or minimum value of a function. Remember that relative max/min value is the y-coordinate of the point.

42. Given $y = x^3 - 15x + 2$

- a) Find the appropriate viewing window to see a complete graph.
- b) Find the x-intercepts, accurate to 3 decimal places.
- c) Find the relative maximum and minimum values, accurate to 3 decimal places

43. Find the coordinates of the intersection points for the functions $f(x) = x + 3$, $g(x) = -x^2 - x + 7$

Section 10: Calculus Skills

Using Khan Academy or other online resources, learn the product, quotient, and chain rules for derivatives. Also, memorize the following derivatives:

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Use these to find the derivatives of each of the following:

$$44. f(x) = (2x+1)(3x-2)$$

$$45. f(x) = \frac{2x+1}{3x-2}$$

$$46. f(x) = x^3 \sin x$$

$$47. f(x) = \frac{\cos x}{4x^5 + 1}$$

$$48. f(x) = 5 \sin 3x$$

$$49. f(x) = \sqrt{\cos x}$$

$$50. f(x) = 5 \sin^2(2x^3 - 4)$$

$$51. f(x) = \sqrt[3]{\cos(x^2)}$$

$$52. f(x) = \frac{2x+3}{(5x-1)^3}$$

Answers

Section 1: Algebra Review

$$1. y = \frac{3x+2}{x-3}$$

$$2. y = x^2 e^x + 1$$

$$3. y = (2x+3)(4x^2 - 6x + 9)$$

$$4. y = 3\sqrt{x}(x - 3 + \frac{6}{x})$$

$$5. y = (x-3)(x+2)(x-2)$$

$$6. 10x + 5h$$

$$7. \frac{x+4}{x(3x-1)}$$

$$8. x = -\frac{2}{3}$$

$$9. (x-1)^2(2x^2 - 7x - 9)$$

$$10. \frac{3x^2-4x-1}{(x-1)^3}$$

$$11. -\frac{1}{3+\sqrt{x+9}}$$

$$12. \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$13. [1,5]$$

$$14. (-\infty, -\frac{5}{2}) \cup (\frac{3}{2}, \infty)$$

$$15. (-2,5)$$

Section 2: Exponentials and Logarithms

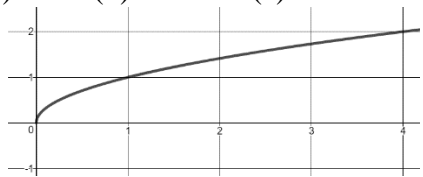
- 16. -3
- 17. 1.631
- 18. 0.763
- 19. 9
- 20. 54.598
- 21. 3
- 22. $\log 3 + 2 \log x + 3 \log y - 4 \log z - \frac{1}{2} \log w$
- 23. $\ln \frac{xz}{w^4 \sqrt[3]{y}}$
- 24. 5t
- 25. $\left(\frac{3}{4}\right)^{\frac{t}{10}}$

Section 3: Trig Review

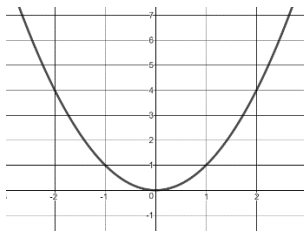
- 26. $\sin^2 \theta + \cos^2 \theta = 1$; $\tan^2 \theta + 1 = \sec^2 \theta$; $\cot^2 \theta + 1 = \csc^2 \theta$
- 27. $\cos 2x = (\cos x)^2 - (\sin x)^2$ or $2(\cos x)^2 - 1$ or $1 - 2(\sin x)^2$
 $\sin 2x = 2 \sin x \cos x$
- 28. $(\sin x)^2 = \frac{1 - \cos 2x}{2}$; $(\cos x)^2 = \frac{1 + \cos 2x}{2}$
- 29. (a) $-\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) 1 (e) -1 (f) -1 (g) $\frac{1}{2}$ (h) $\frac{\sqrt{3}}{2}$ (i) $-\frac{\sqrt{3}}{2}$ (j) $\frac{\pi}{3}$ (k) $\frac{2\pi}{3}$ (l) $-\frac{\pi}{4}$

30. (a) odd (b) neither (c) even

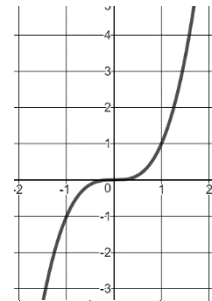
31. a



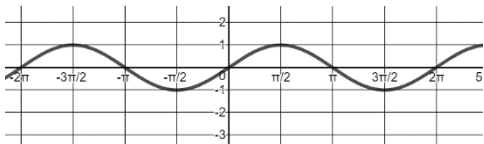
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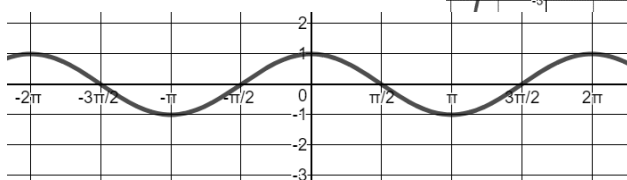
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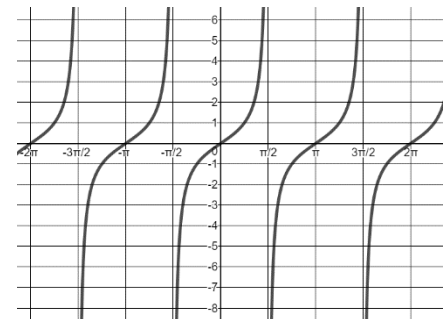
d.



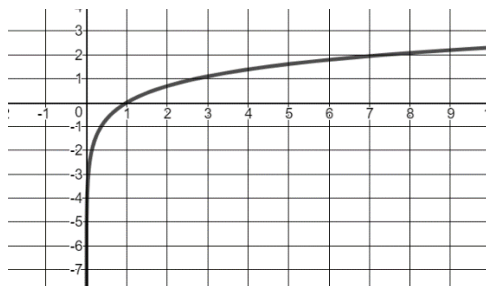
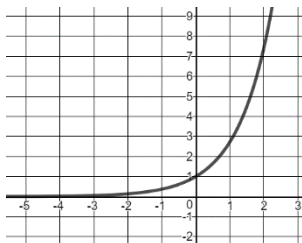
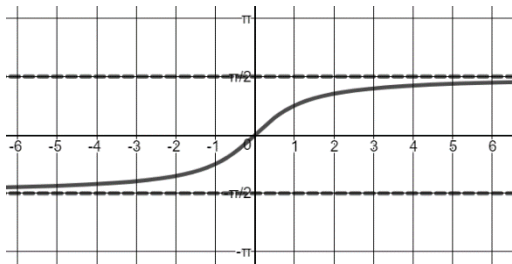
e.



f.



g.



32. $y - 5 = -\frac{2}{3}(x - 3)$

33. $y + 3 = -\frac{2}{3}(x - 2)$ or $y - 1 = -\frac{2}{3}(x + 4)$

34. $y = 8$

35. 50 mph

36. 14

37.

38.

39. $y = \frac{2}{x+5} - 1$

40. (b, a)

41. $x = 1$

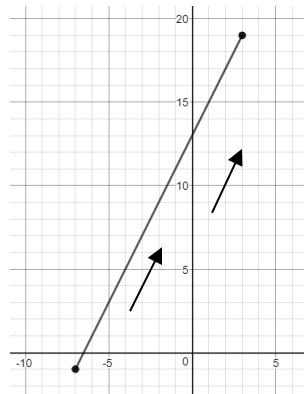
42. (a) $x = [-6, 6]$, $y = [-30, 30]$

(b) $(-3.938, 0), (0.133, 0), (3.805, 0)$

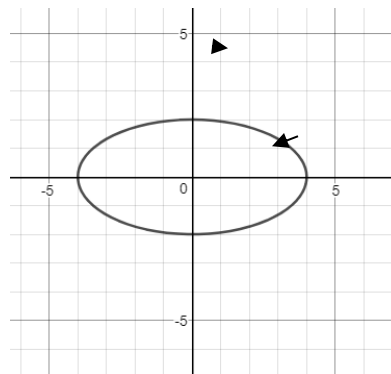
(c) rel max: 24.361, rel min: -20.361

43. $(-3.236, -0.236), (1.236, 4.236)$

#37



#38



44. $12x - 1$

45. $\frac{-7}{(3x-2)^2}$

46. $x^3 \cos x + 3x^2 \sin x$

47. $\frac{(-4x^5 - 1)\sin x - 20x^4 \cos x}{(4x^5 + 1)^2}$

48. $15 \cos 3x$

49. $\frac{-\sin x}{2\sqrt{\cos x}}$

50. $60x^2 \sin(2x^3 - 4)\cos(2x^3 - 4)$

51. $\frac{-2x \sin(x^2)}{3(\cos(x^2))^{\frac{2}{3}}}$

52. $\frac{2(5x-1)^3 - 15(2x+3)(5x-1)^2}{(5x-1)^6}$ which you can simplify by taking out a common factor of $(5x-1)^2$

in the numerator and then simplify to $\frac{-20x-47}{(5x-1)^4}$