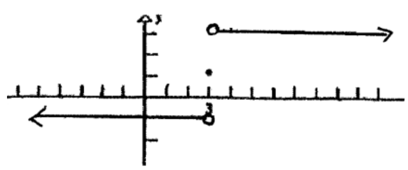
B1 Side 1   Limits

1. For the function *f* graphed below, find

(a) $\lim_{x\to 3^-} f(x)$ (b) $\lim_{x\to 3^+} f(x)$

(c) $\lim_{x\to 3} f(x)$ (d) $f(3)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$

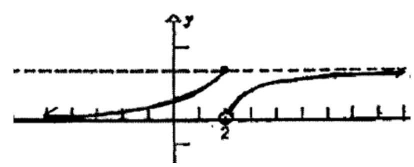


2. For the function *f* graphed below, find

(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$

(c) $\lim_{x\to 2} f(x)$ (d) $f(2)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$

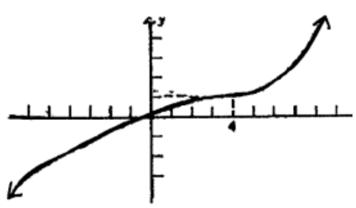


3. For the function *f* graphed below, find

(a) $\lim_{x\to 4^-} f(x)$ (b) $\lim_{x\to 4^+} f(x)$

(c) $\lim_{x\to 4} f(x)$ (d) $f(4)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$

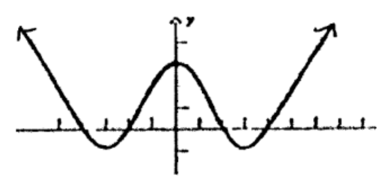


4. For the function *f* graphed below, find

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$

(c) $\lim_{x\to 0} f(x)$ (d) $f(0)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$

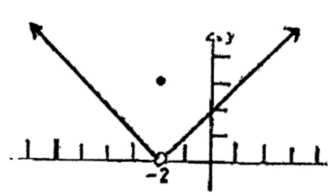


5. For the function *f* graphed below, find

(a) $\lim_{x\to -2^-} f(x)$ (b) $\lim_{x\to -2^+} f(x)$

(c) $\lim_{x\to -2} f(x)$ (d) $f(-2)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$

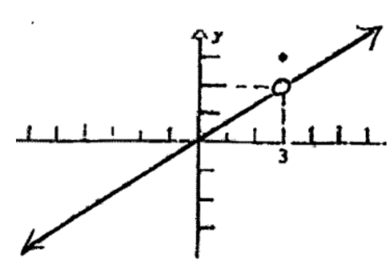


6. For the function *f* graphed below, find

(a) $\lim_{x\to 3^-} f(x)$ (b) $\lim_{x\to 3^+} f(x)$

(c) $\lim_{x\to 3} f(x)$ (d) $f(3)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to \infty} f(x)$



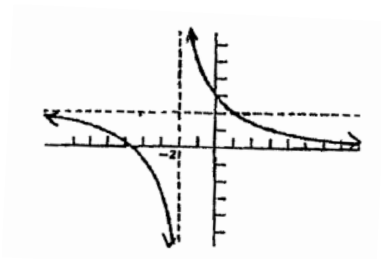
B1 Side 2

7. For the function *f* graphed below, find

(a) $\lim_{x \to -2^-} f(x)$ (b) $\lim_{x \to -2^+} f(x)$

(c) $\lim_{x \to -2} f(x)$ (d) $f(-2)$

(e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to \infty} f(x)$

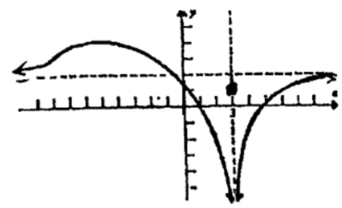


8. For the function F graphed below, find

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$

(c) $\lim_{x \to 3} f(x)$ (d) $f(3)$

(e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to \infty} f(x)$

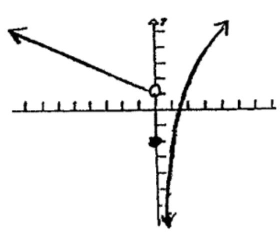


9. For the function *f* graphed below, find

(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$

(c) $\lim_{x \to 0} f(x)$ (d) $f(0)$

(e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to \infty} f(x)$

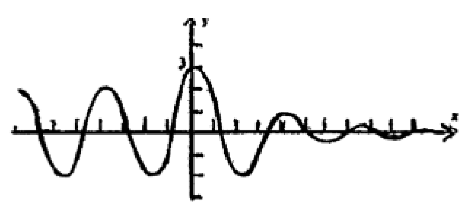


10. For the function *f* graphed below, find

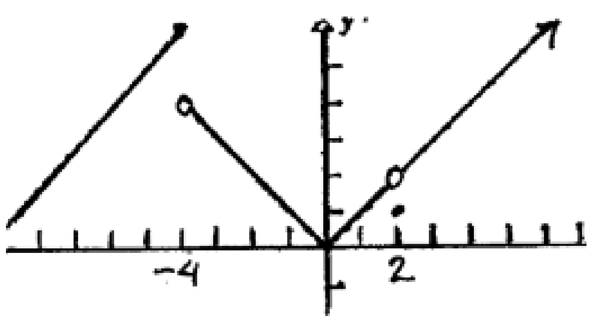
(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$

(c) $\lim_{x \to 0} f(x)$ (d) $f(0)$

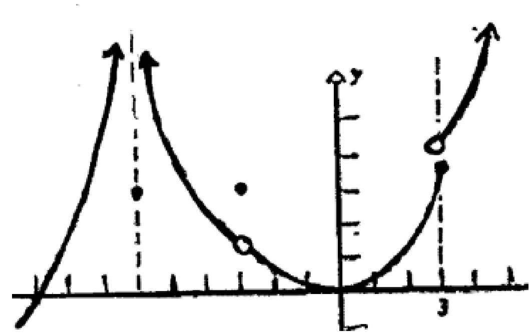
(e) $\lim_{x \to -\infty} f(x)$ (f) $\lim_{x \to \infty} f(x)$



11. Consider the function *g* in the following graph. For what values of x_0 does $\lim_{x \to x_0} g(x)$ exist?



12. Consider the function *f* graphed below. For what values of x_0 does $\lim_{x \to x_0} f(x)$ exist?



B2 Trig limits

Find the following limits. For each problem, you will need to manipulate the given expression so that you can use $\lim_{x\to 0}\frac{\sin x}{x}=1$

1. $\lim_{x\to 0}\frac{\tan x}{x}$

2. $\lim_{x\to 0}\frac{\sin 2x}{x}$

3. $\lim_{x\to 0}\frac{x}{\sin x}$

4. $\lim_{x\to 0}\frac{\sin 5x}{\sin 3x}$ (hint: rewrite as $\frac{\frac{\sin 5x}{x}}{\frac{\sin 3x}{x}}$)

5. $\lim_{x\to 0}\frac{\sin^2 x}{x}$

6. $\lim_{x\to 0}\frac{1-\cos x}{x}$

7. $\lim_{x\to 0}\frac{8x}{\sin 6x}$

8. $\lim_{x\to 0} x\cot 2x$

9. $\lim_{x\to 0}\frac{x+\sin x}{2x}$

10. $\lim_{x\to 0}\frac{\sin x}{2x^2-x}$

B3 Side 1

A.

1. $\lim_{x\to 3} \frac{5x+11}{\sqrt{x+1}}$

2. $\lim_{x\to -2} \frac{6-7x}{(3+2x)^4}$

3. $\lim_{x\to -2} (2x - \sqrt{4x^2 + x})$

4. $\lim_{x\to 4^-} (x - \sqrt{16 - x^2})$

5. $\lim_{x\to 3/2} \frac{2x^2+x-6}{4x^2-4x-3}$

6. $\lim_{x\to 2} \frac{3x^2-x-10}{x^2-x-2}$

7. $\lim_{x\to 2} \frac{x^4-16}{x^2-x-2}$

8. $\lim_{x\to 3^+} \frac{1}{x-3}$

9. $\lim_{x\to 0^+} \frac{1}{\sqrt{x}}$

10. $\lim_{x\to 5} \frac{(1/x)-(\frac{1}{5})}{x-5}$

11. $\lim_{x\to 1/2} \frac{8x^3-1}{2x-1}$

12. $\lim_{x\to 2} 5$

13. $\lim_{x\to 3^+} \frac{3-x}{|3-x|}$

14. $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

15. $\lim_{h\to 0} \frac{(a+h)^4 - a^4}{h}$

16. $\lim_{x\to -3} \sqrt[3]{\frac{x+3}{x^3+27}}$

17. $\lim_{h\to 0} \frac{(2+h)^{-3} - 2^{-3}}{h}$

18. $\lim_{x\to 5} (x^2 + 3)^0$

19. $\lim_{x\to 2^+} \frac{|x-2|}{2-x}$

20. $\lim_{x\to 1} \frac{x-1}{\sqrt{(x-1)^2}}$

Find the limits where [ ] denotes the greatest integer function.

21. $\lim_{x\to 3^+} ([x] - x^2)$

22. $\lim_{x\to 3^-} ([x] - x^2)$

23. Prove, directly from the definition of limit, that $\lim_{x\to 6}(5x - 21) = 9$.

24. Suppose f(x) = 1 if x is rational and f(x) = -1 if x is irrational. Prove that $\lim_{x\to a} f(x)$ does not exist for any real number a.

Find all numbers for which f is continuous.

25. $f(x) = 2x^4 - \sqrt[3]{x} + 1$

26. $f(x) = \sqrt{(2 + x)(3 - x)}$

27. $f(x) = \frac{\sqrt{9 - x^2}}{x^4-16}$

28. $f(x) = \frac{\sqrt{x}}{x^2-1}$

Find the discontinuities of f.

29. $f(x) = \frac{|x^2-16|}{x^2-16}$

30. $f(x) = \frac{1}{x^2-16}$

31. $f(x) = \frac{x^2-x-2}{x^2-2x}$

32. $f(x) = \frac{x+2}{x^3-8}$

33. If $f(x) = 1/x^2$, verify the Intermediate Value Theorem (2.37) for f on $[2, 3]$.

34. Prove that $x^5 + 7x^2 - 3x - 5 = 0$ has a root between -2 and -1.

B3 Side 2

B.

Find the limit (if it exists).

1. $\lim_{x\to 0^+}(4+\sqrt{x})$

2. $\lim_{x\to 0^+}(4x^{3/2}-\sqrt{x}+3)$

3. $\lim_{x\to -6^+}(\sqrt{x+6}+x)$

4. $\lim_{x\to 5/2^-}(\sqrt{5-2x}-x^2)$

5. $\lim_{x\to 5^+}(\sqrt{x^2-25}+3)$

6. $\lim_{x\to 3^-} x\sqrt{9-x^2}$

7. $\lim_{x\to 3^-}\frac{|x-3|}{x-3}$

8. $\lim_{x\to -10^+}\frac{x+10}{|x+10|}$

9. $\lim_{x\to 3^+}\frac{\sqrt{(x-3)^2}}{x-3}$

10. $\lim_{x\to -10^-}\frac{x+10}{\sqrt{(x+10)^2}}$

11. $\lim_{x\to 5^+}\frac{1+\sqrt{2x-10}}{x+3}$

12. $\lim_{x\to 4^+}\frac{\sqrt[4]{x^2-16}}{x+4}$

13. $\lim_{x\to -7^+}\frac{x+7}{|x+7|}$

14. $\lim_{x\to \pi^-}\frac{|\pi-x|}{x-\pi}$

15. $\lim_{x\to 0^+}\frac{1}{x}$

16. $\lim_{x\to 8^-}\frac{1}{x-8}$

For each f, find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$, and sketch the graph of f.

17. $f(x)=\begin{cases}3x & if\ x\le 2\\ x^2 & if\ x>2\end{cases}$

18. $f(x)=\begin{cases}x^3 & if\ x\le 2\\ 4-2x & if\ x>2\end{cases}$

B4 Side 1 Continuity

Definition #1
A function $f(x)$ is continuous at $x = a$ iff all of the following conditions are met:

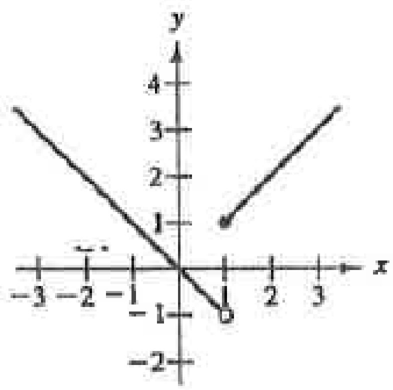
1. $f(a)\ is\ defined$
2. $\lim_{x \to a} f(x) = L, L\ a\ real\ number$
3. $f(a) = L$

Definition #2
A function f is continuous on the interval $[a, b]$ if f is continuous at every number in the interval (a, b) and if the two one sided limits at a and b, respectively, agree with $f(a)$ and $f(b)$, respectively.

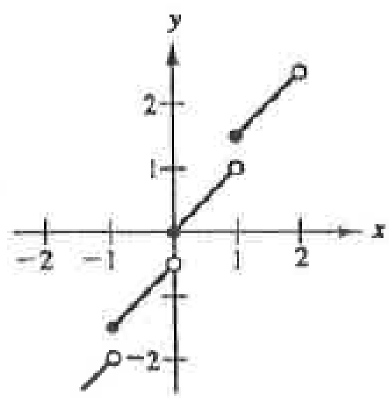
Investigate the continuity of each function for the given value of a. Justify your answers using the above definitions.

1. $f(x) = \begin{cases} 1 - x & x \leq 2 \\ x^2 - 2x & x > 2 \end{cases}$ $\quad a = 2$

17. $f(x) = \begin{cases} -x, & x < 1 \\ 1, & x = 1 \\ x, & x > 1 \end{cases}$ $\quad a = 1$



18. $f(x) = \frac{[\![x]\!]}{2} + x$



19. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2 & x \geq 1 \end{cases}$ $\quad a = 1$

20. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x \leq 2 \\ 3 - x & x > 2 \end{cases}$ $\quad a = 2$

21. $f(x) = \begin{cases} 3 + x, & x \leq 2 \\ x^2 + 1 & x > 2 \end{cases}$ $\quad a = 2$

22. $f(x) = \begin{cases} |x - 2| + 3, & x < 0 \\ x + 5, & x \geq 0 \end{cases}$ $\quad a = 0$

B4 Side 2

23. $f(x) = \frac{|x+1|}{x+1} \quad a = -1$

24. $f(x) = \frac{|4-x|}{4-x} \quad a = 4$

25. $f(x) = [\![x-1]\!]$

26. $f(x) = x - [\![x]\!]$

27. $h(x) = f(g(x)), \qquad f(x) = \frac{1}{\sqrt{x}}, \qquad g(x) = x - 1, x > 1$

28. $h(x) = f(g(x)), \qquad f(x) = \frac{1}{x-1}, \qquad g(x) = x^2 + 5$

39. Determine the constant a such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$$

40. Determine the constants a and b such that the function is continuous on the entire real line.

$$f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$$

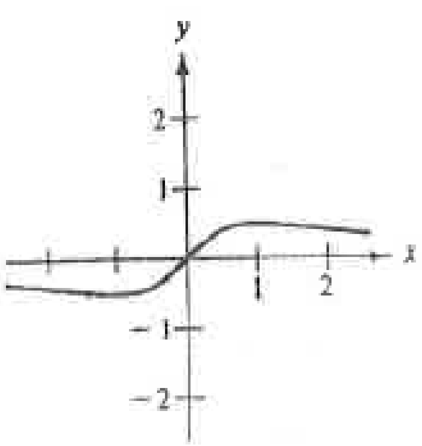
In Exercises 41 and 42, use a graphing utility to graph the function. Then use the graph to determine any x-values at which the function is not continuous.

41. $h(x) = \frac{1}{x^2 - x - 2}$

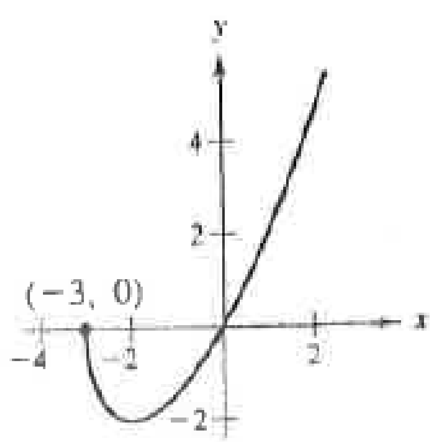
42. $f(x) = \begin{cases} 2x - 4, & x \le 3 \\ x^2 - 2x, & x > 3 \end{cases}$

In Exercises 43-46, find the interval(s) on which the function is continuous.

43. $f(x) = \frac{x}{x^2+1},$



44. $f(x) = x\sqrt{x+3}$



B5 top — Difference Quotient

Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ or $\frac{f(x)-f(a)}{x-a}$ for the following.

1. $f(x)=2x^2-3x+4$
2. $f(x)=x^3+2x-1$
3. $f(x)=\frac{1}{x-4}$
4. $f(x)=\frac{1}{(x+2)^2}$
5. $f(x)=\frac{x}{x+1}$
6. $f(x)=\sqrt{3x-4}$

B5 bottom — Intermediate Value Theorem

For each of the following problems:
A) Decide whether or not the Intermediate Value Theorem can be applied to the given function, value and interval. Justify your decision carefully.
B) When this theorem applies, state carefully what it concludes.
C) Finally, find all values *c* in the given interval, such that $f(c)=k$.

1. $f(x)=x^2-2x+1;\ \ k=4, [-3,2]$
2. $f(x)=x^2+x+1\ ; k=7, [-2,3]$
3. $f(x)=\frac{10}{x^2+1}\ \ ; k=8, [0,1]$
4. $f(x)=\sqrt{x+3}\ \ ; k=2, [-3,6]$
5. $f(x)=\frac{1}{2x-1}\ \ ; k=0.5, [1,2]$
6. $f(x)=\frac{x^2+8}{x}\ \ ; k=6, [1,3]$

B6 side 1

1. In each part, find the limit by inspection.

(a) $\lim_{x\to -\infty}(-3)$ (b) = $\lim_{h\to +\infty}(-2h)$

2. In each part, find the stated limit of $f(x) = x/|x|$ by inspection.

(a) $\lim_{x\to +\infty} f(x)$ (b) $\lim_{x\to -\infty} f(x)$

3. Given that

$$\lim_{x\to +\infty} f(x) = 3, \qquad \lim_{x\to +\infty} g(x) = -5, \qquad \lim_{x\to +\infty} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to +\infty}[f(x) + 3g(x)]$ (b) $\lim_{x\to +\infty}[h(x) - 4g(x) + 1]$

(c) $\lim_{x\to +\infty}[f(x)g(x)]$ (d) $\lim_{x\to +\infty}[g(x)]^2$

(e) $\lim_{x\to +\infty}\sqrt[3]{5 + f(x)}$ (f) $\lim_{x\to +\infty}\frac{3}{g(x)}$

(g) $\lim_{x\to +\infty}\frac{3h(x)+4}{x^2}$ (h) $\lim_{x\to +\infty}\frac{6f(x)}{5f(x)+3g(x)}$

4. Given that

$$\lim_{x\to -\infty} f(x) = 7, \qquad \lim_{x\to -\infty} g(x) = -6$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to -\infty}[2f(x) - g(x)]$ (b) $\lim_{x\to -\infty}[6f(x) + 7g(x)]$

(c) $\lim_{x\to -\infty}[x^2 + g(x)]$ (d) $\lim_{x\to -\infty}[x^2 g(x)]$

(e) $\lim_{x\to -\infty}\sqrt[3]{f(x)g(x)}$ (f) $\lim_{x\to -\infty}\frac{g(x)}{f(x)}$

(g) $\lim_{x\to -\infty}[f(x) + \frac{g(x)}{x}]$ (h) $\lim_{x\to -\infty}\frac{xf(x)}{(2x+3)g(x)}$

In Exercises 5-28, find the limits.

5. $\lim_{x\to -\infty}(3 - x)$

6. $\lim_{x\to -\infty}(5 - \frac{1}{x})$

7. $\lim_{x\to +\infty}(1 + 2x - 3x^5)$

8. $\lim_{x\to +\infty}(2x^3 - 100x + 5)$

9. $\lim_{x\to +\infty}\sqrt{x}$

10. $\lim_{x\to -\infty}\sqrt{5 - x}$

B6 Side 2

11. $\lim_{x\to +\infty} \frac{3x+1}{2x-5}$

12. $\lim_{x\to +\infty} \frac{5x^2-4x}{2x^2+3}$

13. $\lim_{y\to -\infty} \frac{3}{y+4}$

14. $\lim_{x\to +\infty} \frac{1}{x-12}$

15. $\lim_{x\to -\infty} \frac{x-2}{x^2+2x+1}$

16. $\lim_{x\to +\infty} \frac{5x^2+7}{3x^2-x}$

17. $\lim_{x\to +\infty} \sqrt[3]{\frac{2+3x-5x^2}{1+8x^2}}$

18. $\lim_{s\to +\infty} \sqrt[3]{\frac{3s^7-4s^5}{2s^7+1}}$

19. $\lim_{x\to -\infty} \frac{\sqrt{5x^2-2}}{x+3}$

20. $\lim_{x\to +\infty} \frac{\sqrt{5x^2-2}}{x+3}$

21. $\lim_{y\to -\infty} \frac{2-y}{\sqrt{7+6y^2}}$

22. $\lim_{y\to +\infty} \frac{2-y}{\sqrt{7+6y^2}}$

23. $\lim_{x\to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$

24. $\lim_{x\to +\infty} \frac{\sqrt{3x^4+x}}{x^2-8}$

25. $\lim_{x\to +\infty} \frac{7-6x^5}{x+3}$

26. $\lim_{t\to -\infty} \frac{5-2t^3}{t^2+1}$

27. $\lim_{t\to +\infty} \frac{6-t^3}{7t^3+3}$

28. $\lim_{x\to -\infty} \frac{x+4x^3}{1-x^2+7x^3}$

29. Let

$$f(x) = \begin{cases} 2x^2+5, & x < 0 \\ \frac{3-5x^3}{1+4x+x^3}, & x \geq 0 \end{cases}$$

Find

(a) $\lim_{x\to -\infty} f(x)$

(b) $\lim_{x\to +\infty} f(x)$

30. Let

$$g(t) = \begin{cases} \frac{2+3t}{5t^2+6}, & t < 1{,}000{,}000 \\ \frac{\sqrt{36t^2-100}}{5-t}, & t > 1{,}000{,}000 \end{cases}$$

Find

(a) $\lim_{t\to -\infty} g(t)$

(b) $\lim_{t\to +\infty} g(t)$

B6 Side 3

In Exercises 31-34, find the limits.

31. $\lim_{x\to +\infty} (\sqrt{x^2+3} - x)$

32. $\lim_{x\to +\infty} (\sqrt{x^2-3x} - x)$

33. $\lim_{x\to +\infty} (\sqrt{x^2+ax} - x)$

34. $\lim_{x\to +\infty} (\sqrt{x^2+ax} - \sqrt{x^2+bx})$

35. Discuss the limits of $p(x) = (1-x)^n$ as $x \to +\infty$ and $x \to -\infty$ for positive integer values of n.

36. Let $p(x) = (1-x)^n$ and $q(x) = (1-x)^m$. Discuss the limits of $p(x)/x^m$ as $x \to +\infty$ and $x \to -\infty$ for positive integer values of m and n.

37. Let $p(x)$ be a polynomial of degree n. Discuss the limits of $p(x)/x^m$ as $x \to +\infty$ and $x \to -\infty$ for positive integer values of m.

38. In each part, find examples of polynomials $p(x)$ and $q(x)$ that satisfy the stated condition and such that $p(x) \to +\infty$ and $q(x) \to +\infty$ as $x \to +\infty$.

(a) $\lim_{x\to +\infty} \frac{p(x)}{q(x)} = 1$

(b) $\lim_{x\to +\infty} \frac{p(x)}{q(x)} = 0$

(c) $\lim_{x\to +\infty} \frac{p(x)}{q(x)} = +\infty$

(d) $\lim_{x\to +\infty} [p(x) - q(x)] = 3$

39. Assuming that m and n are positive integers, find

$$\lim_{x\to -\infty} \frac{2+3x^n}{1-x^m}$$

B7

Definition of Derivative

Show your work on a separate piece of paper.

Use the form: $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find the derivative of each of the following:

1. $f(x)=x^2+2$
2. $f(x)=\frac{1}{\sqrt{x}}$
3. $f(x)=\sqrt{1-x}$
4. $f(x)=\frac{1}{x^2+1}$

Use the form $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ to find the value of each derivative at the given point.

5. $f(x)=x^2$ at (2,4)
6. $f(x)=2x-x^2$ at (1,1)

Using either of the forms above, find the slope of the tangent to $y=f(x)$ at the given point. Then write the equation of the line tangent to $y=f(x)$ at the given point.

7. $f(x)=x^2+x+2$ at (1, 4)
8. $f(x)=\sqrt{x}$ at (9, 3)

B8

Differentiability

For which value or values of *x* does g fail to have a derivative. You may use your graphing calculator to investigate the graphs. If you do, draw a sketch of the graph on your paper.

9. $g(x)=|x|-2$
10. $g(x)=|x-2|$
11. $g(x)=|9-x^2|$
12. $g(x)=|4x-x^2|$

Which of the following are differentiable at *x* = 0? For those that are, find f $f'(0)$. You may use your graphing calculator.

13. $f(x)=x|x|$
14. $f(x)=\sqrt{x^2}$
15. $f(x)=\sqrt{|x|}$
16. $f(x)=x(1-|x|)$

B9 Side 1 **Limits Practice**

1) $\lim_{x\to\infty} \frac{2x+3}{5x+7} =$

2) $\lim_{x\to\infty} \frac{2x^3+7}{x^3-x^2+x+7} =$

3) $\lim_{x\to\infty} \frac{3x+7}{x-2} =$

4) $\lim_{x\to\infty} \frac{3x^2-6x}{4x-8} =$

5) $\lim_{x\to\infty} \frac{7x+1}{x^3+3} =$

6) $\lim_{x\to\infty} \frac{2x^4}{x^3+1} =$

7) $\lim_{x\to\infty} \frac{-x^4}{x^4-7x^3+7x^2+9} =$

8) $\lim_{x\to\infty} \frac{10x^5+10x^4}{x^6+31} =$

9) $\lim_{x\to\infty} \frac{1}{x^3-4x+1} =$

10) $\lim_{x\to\infty} (\frac{-x}{5+x^2})(\frac{x^2}{2x+1}) =$

11) $\lim_{x\to\infty} (\frac{2x+1}{3x-2})(\frac{5x}{6x+1})$

12) $\lim_{x\to 2} \frac{2x+1}{x-1} =$

13) $\lim_{x\to 1} \frac{2x+1}{x+1} =$

14) $\lim_{x\to 3^-} \frac{1}{x-3} =$

15) $\lim_{x\to 3^+} \frac{1}{x-3} =$

16) $\lim_{x\to 1} \frac{x-1}{x^2-1} =$

17) $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-1} =$

18) $\lim_{x\to 2} \frac{2x-4}{x^3-2x^2} =$

19) $\lim_{x\to -5} \frac{x^2+3x-10}{x+5} =$

20) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4} =$

21) $\lim_{x\to 0} \frac{5x^3+8x^2}{3x^4-16x^2} =$

22) $\lim_{x\to 3} \frac{2x-1}{x} =$

23) $\lim_{x\to\infty} -\frac{x}{7x+4} =$

24) $\lim_{x\to +\infty} \frac{1}{x} =$

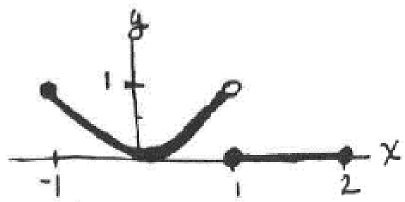
25) $\lim_{x\to -\infty} \frac{1}{x} =$

26) $\lim_{x\to 0} \frac{(x+2)^3-8}{x} =$

27) $\lim_{x\to -1^+} f(x) =$

28) $\lim_{x\to 0^+} f(x) =$

B9 Side 2



29) $\lim_{x\to 1^-} f(x) =$

30) $\lim_{x\to 1^+} f(x) =$

31) $\lim_{x\to 2^-} f(x) =$

32) $\lim_{x\to 0^-} f(x) =$

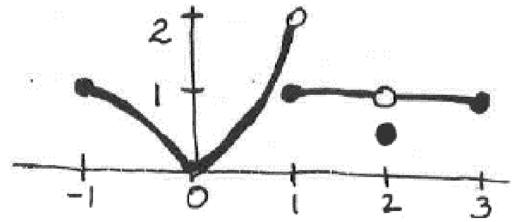
33) $\lim_{x\to 1} f(x) =$

34) $\lim_{x\to 0} f(x) =$

35) $\lim_{x\to 3} \sqrt{x+1} =$

36) $\lim_{x\to 0} \frac{(4+x)^2-16}{x} =$

37) $\lim_{x\to 3}(x-2)^{1998} =$



38) $\lim_{x\to -1^+} g(x) =$

39) $\lim_{x\to 2} g(x) =$

40) $\lim_{x\to 1^-} g(x) =$

41) $\lim_{x\to 0} g(x) =$

42) $\lim_{x\to 1^+} g(x) =$

43) $\lim_{x\to 3^-} g(x) =$

44) $\lim_{x\to 1} g(x) =$

45) $\lim_{x\to 3} 2x =$

46) $\lim_{x\to \frac{1}{2}}(2x-1) =$

47) $\lim_{x\to -1} 3x^2(2x-1) =$

48. $\lim_{x\to -1}(3x-1) =$

B10 Side 1

A

1. $P(1, 1)$ is a point on the graph of $y = x^3$. Find the slope of PQ if Q is:

a. $(2, 8)$ b. $(1.5, 3.375)$ c. $(1.1, 1.331)$

d. If $f(x) = x^3$, find $f'(x)$ and $f'(1)$.

2. $P(2, \frac{1}{2})$ is a point on the graph of $y = \frac{1}{x}$. Find the slope of $\overline{PQ}$ if Q is:

a. $(3, f(3))$ b. $(2.5, f(2.5))$ c. $(2.1, f(2.1))$

d. If $f(x) = \frac{1}{x}$, find $f'(x)$ and $f'(2)$.

Find the derivative of the given function. Express the derivative without using fractional or negative exponents.

3. $f(x) = 2x^5$ 4. $f(x) = 3x^6$ 5. $f(x) = -2x^3$ 6. $f(x) = 8x^{3/4}$

7. $f(x) = \frac{3}{x^2}$ 8. $g(x) = \frac{4}{\sqrt{x}}$ 9. $f(x) = \sqrt{5x}$ 10. $f(x) = \frac{1}{4x^4}$

11. $g(x) = \frac{7}{2}x^2 - 5x + 3 - \frac{1}{x}$ 12. $f(x) = \frac{4}{3}x^3 - \frac{2}{x} - \frac{5}{x^2} + \pi$

Find the slope of each curve at the given point P.

13. $y = x^3; P(-1, -1)$ 14. $y = \frac{x^4}{2}; P(-1, \frac{1}{2})$

15. $y = 3x^2 - 2x + 1; P(0, 1)$ 16. $y = 2\sqrt{x}; P(9, 6)$

17. $y = 8x^4 - 7x^2 + 5x + 6; P(-1, 2)$ 18. $y = x^3 - 5x^2 + 4x + 2; P(2, -2)$

B

19. $y = \frac{2}{x} - \frac{4}{x^2}; P(2, 0)$ 20. $y = \sqrt[3]{x}; P(-1, -1)$

21. a. If $f(x) = ax^2 + bx + c$ where $a \neq 0$, find $f'(x)$.

b. **Visual Thinking** Evaluate $f'(-\frac{b}{2a})$ and explain what the result says about the tangent to the parabola $y = f(x)$ at the vertex

22. **Visual Thinking** Give a geometric argument to explain why two functions whose rules differ only by a constant have the same derivative.

B10 Side 2

Find an equation of the line tangent to the given curve at the given point P.

23. $y = x^3; P(-2, -8)$

24. $y = 3x^2 - 5x; P(2, 2)$

25. $y = \frac{1}{x^2}; P(-1, 1)$

26. $y = \frac{1}{\sqrt[3]{x}}; P(8, \frac{1}{2})$

27. a. Sketch the graph of the function $f(x) = 6x^2 - x^3$
    b. For what values of x does $f'(x) = 0$?
    c. On your graph of $f(x)$, indicate the two points where $f'(x) = 0$.

28. Repeat Exercise 27 using the function $f(x) = x^3 - 9x$.

29. a. If $f(x) = x^{2/3}$, find $f'(x)$.
    b. For what values of x is $f'(x)$ undefiend?
    c. Sketch the graph of $f(x)$. (If you use a computer on graphing calculator, you may need to enter the rule for the function as $(x^2)^{1/3}$ to obtain the complete graph.) Explain how the graph of $f(x)$ supports the result of part (b).

30. Repeat Exercise 29 using the function $f(x) = x^{1/3}$.

In Exercises 31-36, find a function that has the given derivative.

31. $f'(x) = 4x^3$

32. $f'(x) = 5x^4$

33. $g'(x) = 3x^5$

34. $g'(x) = x^7$

35. $h'(x) = 3\sqrt{x}$

36. $h'(x) = \sqrt[3]{x}$

C

37.

a. Use the binomial theorem (page 591) to write the first three terms and the last term in the expansion of $(x + h)^n$.

b. Use part (a) and the definition of $f'(x)$ to prove Theorem 2 on page 760 for positive integral values of n.

(Note: The binomial theorem can be generalized to apply to any nonzero real number n. This form of the binomial theorem can then be used to give a general proof of Theorem 2.)

38. Prove Theorem 3 on page 760. (Hint: Use the definition of $f'(x)$.)

39. Prove Theorem 4 on page 760. (Hint: Use the definition of $f'(x)$.)

B11

Sketching Polynomials

Find the local extrema of f. Describe the intervals in which f is increasing or decreasing and sketch the graph of f.

1. $f(x)=5-7x-4x^2$
2. $f(x)=6x^2-9x+5$
3. $f(x)=2x^3+x^2-20x+1$
4. $f(x)=x^3-x^2-40x+8$
5. $f(x)=x^4-8x^2+1$
6. $f(x)=x^3-3x^2+3x+7$
7. $f(x)=x^{4/3}+4x^{1/3}$
8. $f(x)=x^{2/3}(8-x)$
9. $f(x)=x^2\sqrt[3]{x^2-4}$
10. $f(x)=x\sqrt{4-x^2}$
11. $f(x)=x^{2/3}(x-7)^2+2$
12. $f(x)=4x^3-3x^4$
13. $f(x)=x^3+(3/x)$
14. $f(x)=8-\sqrt[3]{x^2-2x+1}$
15. $f(x)=10x^3(x-1)^2$
16. $f(x)=(x^2-10x)^4$

Find the local extrema of f.

17. $f(x)=\sqrt[3]{x^3-9x}$
18. $f(x)=x^2/\sqrt{x+7}$
19. $f(x)=(x-2)^3(x+1)^4$
20. $f(x)=x^2(x-5)^4$
21. $f(x)=(2x-5)/(x+3)$
22. $f(x)=(x^2+3)/(x-1)$

23-26 Find the absolute maximum and minimum values on each of the given intervals:

23. $f(x)=5-7x-4x^2$
(a) [-1, 1] (b)[-4, 2] (c)[0, 5]

24. $f(x)=6x^2-9x+5$
(a) [-1, 1] (b)[-4, 2] (c)[0, 5]

25. $f(x)=2x^3+x^2-20x+1$
(a) [-1, 1] (b)[-4, 2] (c)[0, 5]

26. $f(x)=x^3-x^2-40x+8$
(a) [-1, 1] (b)[-4, 2] (c)[0, 5]

Sketch the graph of a differentiable function f which satisfies the given conditions.

27. $f'(-5)=0$; $f'(0)=0$; $f'(5)=0$; $f'(x)>0$ If $|x|>5$; $f'(x)<0$ if $0<|x|<5$

28. $f'(a)=0$ for $a=1,2,3,4,5$ and $f'(x)>0$ for all other values of x.

Find the local extrema of f. Describe the intervals in which f' is increasing or decreasing. Sketch the graph of f and study the variation of the slope of the tangent line as x increases through the domain of f.

29. $f(x)=x^4-6x^2$
30. $f(x)=4x^3-3x^4$

31. If $f(x)=ax^3+bx^2+cx+d$, determine Values for a, b, c, and d such that f has a local Maximum 2 at x=-1 and a local minimum -1 at *x*=1.

32. If $f(x)=ax^4+bx^3+cx^2+dx+e$, determine values of a, b, c, d and e such that f has a local maximum 2 at *x*=0, and a local minimum -14 at x=-2 and x=2, respectively.

B12 Side 1

Concavity

Use the Second Derivative Test (whenever applicable) to find the local extrema of f . Discuss concavity, find x-coordinates of points of inflection, and sketch the graph of f .

1. $f(x)=x^3-2x^2+x+1$
2. $f(x)=x^3+10x^x+25x-50$
3. $f(x)=3x^4-4x^3+6$
4. $f(x)=8x^2-2x^4$
5. $f(x)=2x^6-6x^4$
6. $f(x)=3x^5-5x^3$
7. $f(x)=(x^2-1)^2$
8. $f(x)=x-(16/x)$
9. $f(x)=\sqrt[5]{x}-1$
10. $f(x)=(x+4)/\sqrt{x}$
11. $f(x)=x^2-(27/x^2)$
12. $f(x)=x^{2/3}(1-x)$
13. $f(x)=x/(x^2+1)$
14. $f(x)=x^2/(x^2+1)$
15. $f(x)=\sqrt[3]{x^2}(3x+10)$
16. $f(x)=x^4-4x^3+10$
17. $f(x)=8x^{1/3}+x^{4/3}$
18. $f(x)=x\sqrt{4-x^2}$

Sketch the graph of a continuous function f which satisfies all of the started conditions.

19. $f(0)=1$; $f(2)=3$;
$f'(0)=f'(2)=0$;
$f'(x)<0$ if $|x-1|>1$;
$f'(x)>0$ if $|x-1|<1$;
$f''(x)>0$ if $x<1$;
$f''(x)<0$ if $x>1$

20. $f(0)=4$; $f(2)=2$; $f(5)=6$
$f'(0)=f'(2)=0$
$f'(x)>0$ if $|x-1|>1$;
$f'(x)<0$ if $|x-1|<1$;
$f''(x)<0$ if $x<1$ or if $|x-4|<1$;
$f''(x)>0$ if $|x-2|<1$ or if $x>5$

21. $f(0)=2$; $f(2)=f(-2)=1$;
$f'(0)=0$;
$f'(x)>0$ if $x<0$;
$f'(x)<0$ if $x>0$;

$f''(x)<0$ if $|x|<2$;
$f''(x)>0$ if $|x|>2$.

22. $f(1)=4$;
$f'(x)>0$ if $x<1$;
$f'(x)<0$ if $x>1$;
$f''(x)>0$ for all $x\neq 1$.

23. $f(-2)=f(6)=-2$; $f(0)=f(4)=0$;
$f(2)=f(8)=3$;
f' is undefined at 2 and 6;
$f'(0)=1$;
$f'(x)>0$ throughout $(-\infty,2)$ and $(6,\infty)$;
$f'(x)<0$ if $|x-4|<2$;
$f''(x)<0$ throughout $(-\infty,0)$, $(4,6)$, $(6,\infty)$;
$f''(x)>0$ throughout $(0,2)$ and $(2,4)$.

24. $f(0)=2$; $f(2)=1$; $f(4)=f(10)=0$; $f(6)=-4$
$f'(2)=f'(6)=0$;
$f'(x)<0$ for x in $(-\infty,2)$, $(2,4)$, $(4,6)$, and $(10,\infty)$;
$f'(x)>0$ throughout $(6,10)$;
$f'(4)$ and $f'(10)$ do not exist;
$f''(x)>0$ throughout $(-\infty,2)$, $(4,10)$, and $(10,\infty)$;
$f''(x)<0$ throughout $(2,4)$

B12 Side 2

25. If n is an odd integer, then $f(n)=1$ and $f'(n)=0$; if n is an even integer, then $f(n)=0$ and $f'(n)$ does not exist; if n is any integer then
(a) $f'(x)>0$ whenever $2n<x<2n+1$
(b) $f'(x)<0$ whenever $2n-1<x<2n$
(c) $f''(x)<x$ whenever $2n<x<2n+2$

26. $f(x)=x$ if x= -1, 2, 4 or 8;
$f'(x)=0$ if x= -1, 4, 6 or 8;
$f'(x)<0$ throughout $(-\infty,-1)$, (4, 6), and $(8,\infty)$;
$f'(x)>0$ throughout (-1, 4) and (6, 8);
$f''(x)>0$ throughout ($(-\infty,0)$, (2, 3), and (5, 7);
$f''(x)<0$ throughout (0, 2), (3, 5) and $(7,\infty)$.

27. Prove that the graph of a quadratic function has no points of inflection. State conditions under which the graph is always (a) concave upward; (b)concave downward. Illustrate with sketches.

28. Prove that the graph of a polynomial function of degree 3 has exactly one point of inflection. Illustrate this fact with sketches.

29. Prove that the graph of a polynomial function of degree n>2 has at most n-2 points of inflection.

30. If $f(x)=x^n$, where n>1, prove that the graph of f has either one or no points of inflection, according to whether n is odd or even. Illustrate with sketches.

B13

Justifying answers

1. $f(x) = 2x^4 - 4x^3$ Find all extrema. Justify

a) using the first derivative test.

b) using the second derivative test

2. $f(x) = 2x^6 - 6x^4$ Find all extrema. Justify

a) using the first derivative test

b) using the second derivative test.

3. Find the absolute maximum and absolute minimum of $f(x) = 2x^6 - 6x^4$ over the interval [-2, 1]. Justify your answer using candidates test.

B14 Product & Quotient Rules

6. Differentiate the function: $h(x) = (x - 2)(2x + 3)$

21. Find the derivative of $y = (x^2 + 1)(x^3 + 1)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

22. Find the derivative of the function $F(x) = \frac{x-3x\sqrt{x}}{\sqrt{x}}$ in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method did you prefer?

23 - 42 Differentiate.

23. $V(x) = (2x^3 + 3)(x^4 - 2x)$

25. $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3)$

26. $y = \sqrt{x}(x - 1)$

28. $f(t) = \frac{2t}{4+t^2}$

30. $y = \frac{x+1}{x^3+x-2}$

32. $y = \frac{t}{(t-1)^2}$

34. $g(t) = \frac{t-\sqrt{t}}{t^{4/3}}$

36. $y = A + \frac{B}{x} + \frac{C}{x^2}$

38. $y = \frac{cx}{1+cx}$

40. $y = \frac{u^6-2u^3+5}{u^2}$

42. $f(x) = \frac{ax+b}{cx+d}$

49. Find an equation of the tangent line to the curve at the given point. $y = \frac{2x}{x+1}$, (1,1)

55. Find equations of the tangent line and normal line to the curve $y = \frac{3x+1}{x^2+1}$ at the point (1,2)

64. Find $h'(2)$, given that $f(2) = -3, g(2) = 4, f'(2) = -2, and\ g'(2) = 7.$

(a) $h(x) = 5f(x) - 4g(x)$

(b) $h(x) = f(x)g(x)$

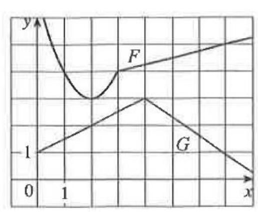
(c) $h(x) = \frac{f(x)}{g(x)}$

(d) $h(x) = \frac{g(x)}{1+f(x)}$

65. If $f(x) = \sqrt{x} \cdot g(x)$, where $g(4) = 8$ and $g'(4) = 7$, find $f'(4)$.

66. If $h(2) = 4$ and $h'(2) = -3$, find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2}$

68. Let $P(x) = F(x)G(x)$ and $Q(x) = F(x)/G(x)$, where F and G are the functions whose graphs are shown. (a) Find $P'(2)$ and (b) Find $Q'(7)$



69. If g is a differentiable function find an expression for the derivative of each of the following functions.

(a) $y = xg(x)$ (b) $y = \frac{x}{g(x)}$ (c) $y = \frac{g(x)}{x}$

71. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

72. For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent?

74. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$.

76. Find equations of the tangent lines to the curve $y = \frac{x-1}{x+1}$ that are parallel to the line $x - 2y = 2$

B15 Review Limits/Functions

Find each Limit

1. $\lim_{x\to -3} \frac{2x^2+5x-3}{x^2-x-12}$

2. $\lim_{x\to 0} \frac{3\sin 4x}{\sin 3x}$

3. $\lim_{x\to 0} \frac{\sin x}{3x^2+2x}$

4. $\lim_{x\to\infty} \left(\frac{3x^2+2}{2x^2+1}\right)\left(\frac{1}{x}+5\right)$

5. $\lim_{x\to 3} \frac{5x+11}{\sqrt{x+1}}$

6. $\lim_{x\to 1/2} \frac{8x^3-1}{2x-1}$

7. $\lim_{x\to 2^-} \frac{x-2}{\sqrt{(x-2)^2}}$

8. $\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3}$

9. $\lim_{x\to 5} \frac{(1/x)-(1/5)}{x-5}$

10. $\lim_{x\to 5/2^-} \left(\sqrt{5-2x}-x^2\right)$

11. $\lim_{x\to\infty} \frac{x-10}{\sqrt{3x^2+10}}$

12. $\lim_{x\to 3/5^-} \frac{1}{5x-3}$

13. $f(x)=\begin{cases} x^2 & if\ x<1 \\ 2 & if\ x=1 \\ 4-x^2 & if\ x>1 \end{cases}$

Find $\lim_{x\to 1^+} f(x)=$

$\lim_{x\to 1^-} f(x)=$

$\lim_{x\to 1} f(x)=$

14. Find the Domain/intercepts/asymptotes and graph

a) $f(x)=\sqrt{(2+x)(3-x)}$

b) $f(x)=\frac{6x^2-2x^3}{9-x^2}$

c) $f(x)=\frac{x+2}{x^2+2x-8}$

15. Use the definition of continuity to determine whether f is cont or discont at $x=a$

a) $f(x)=\begin{cases} x^2+1 & if\ x<1 \\ 1 & if\ x=1 \\ x+1 & if\ x>1 \end{cases}$ at $a=1$

b) $f(x)=\frac{\sqrt[3]{x}}{2x+1}$ at $a=8$

16. If $f(x)=\begin{cases} k^2x & if\ x<1 \\ 3kx-2 & if\ x\geq 1 \end{cases}$ find the value of k such that $f(x)$ is continuous.

B16 Review of Derivatives

1. Use the definition of derivative to find f'(x).
   a) $f(x)=\sqrt{3x-1}$ b) $f(x)=x^2-3x$

2. Use the definition of derivative to find the slope of the tangent line to $f(x)=\dfrac{1}{2x+3}$ at $x=3$.

Use the power rule now

3. Find the equation of the tangent line to $f(x)=2x^3-x^2+5x$ at $x=2$.
4. Find the equation of the line tangent to $f(x)=x-\dfrac{1}{\sqrt{x}}$ at $x=1$.
5. Find the points on $f(x)$ where the tangent line is horizontal.
   a) $f(x)=3x^2-2x+8$ b) $f(x)=3x^4-2x^2+1$

6. Determine whether *f* has a vertical tangent at (0, 0).
   a) $f(x)=x^{\frac{1}{3}}$ b) $f(x)=5x^{\frac{3}{2}}$

7. Find $f'(x)$
   a) $f(x)=3x^{\frac{2}{3}}-5x^3+8$ b) $f(x)=\sqrt{x}-4x^2+\dfrac{2}{x}$ c) $f(x)=e^2$

8. If $f(x)=5x^{\frac{4}{3}}-7x^3+\dfrac{4}{x^2}+5x-6$, find $f'(x),\ f''(x),\ f'''(x)$

B17 Side 1

In Exercises 1-28, evaluate the given limit or state that it does not exist.

A

1. $\lim_{x\to\infty} \frac{3x-5}{4x+9}$

2. $\lim_{x\to\infty} \frac{2x^2-7x}{3x^2+5}$

3. $\lim_{x\to-\infty} \frac{8x^2-7x+5}{4x^2+9}$

4. $\lim_{x\to-\infty} \frac{5x^3}{7x^3+8x^2}$

5. $\lim_{x\to\infty} \frac{(x^2+1)(x^2-1)}{2x^4}$

6. $\lim_{x\to\infty} \frac{x^2\cos\frac{1}{x}}{2x^2-1}$

7. $\lim_{x\to1} \frac{x^2-1}{x-1}$

8. $\lim_{x\to3} \frac{2x^2-6x}{x-3}$

9. $\lim_{x\to2} \frac{x^2+x-6}{2x-4}$

10. $\lim_{x\to4} \frac{x-4}{x^2-x-12}$

11. $\lim_{x\to0^+} \frac{x+1}{x}$

12. $\lim_{x\to-1^+} \frac{x}{x+1}$

13. $\lim_{x\to3^+} \frac{x-4}{x-3}$

14. $\lim_{x\to3^-} \frac{2x-6}{x^2-3x}$

15. $\lim_{x\to-2^-} \frac{x^2+4x+4}{x^2+3x+1}$

16. $\lim_{x\to-2^+} \frac{3x+6}{2x+4}$

17. a. $\lim_{x\to0^+} \frac{|x|}{x}$ b. $\lim_{x\to0^-} \frac{|x|}{x}$ c. $\lim_{x\to0} \frac{|x|}{x}$

18. a. $\lim_{x\to1^+} \frac{x-3}{x^2-1}$ b. $\lim_{x\to1^-} \frac{x-3}{x^2-1}$ c. $\lim_{x\to1} \frac{x-3}{x^2-1}$

19. a. $\lim_{x\to2^+} \frac{x-2}{\sqrt{x^2-4}}$ b. $\lim_{x\to2^-} \frac{x-2}{\sqrt{x^2-4}}$ c. $\lim_{x\to2} \frac{x-2}{\sqrt{x^2-4}}$

20. a. $\lim_{x\to3^+} \lfloor x\rfloor$ b. $\lim_{x\to3^-} \lfloor x\rfloor$ c. $\lim_{x\to3} \lfloor x\rfloor$
(Note: $\lfloor x\rfloor$ is the greatest integer function. See Exercise 21 on page 124).

21. $\lim_{x\to0} \frac{x}{1-\sqrt{1-x}}$

22. $\lim_{x\to0} \frac{2-\sqrt{4-x}}{x}$

23. $\lim_{x\to0} \frac{1-\sqrt{x^2+1}}{x^2}$

24. $\lim_{x\to-\infty} (\sqrt{4x^2-4x}-2x)$

25. $\lim_{x\to\infty} (\sqrt{x+1} - \sqrt{x})$

26. $\lim_{x\to\infty} (\sqrt{x^2+2x}-x)$

27. a. $\lim_{h\to0} \frac{(1+h)^2-1}{h}$ b. $\lim_{h\to0} \frac{(2+h)^2-4}{h}$ c. $\lim_{h\to0} \frac{(x+h)^2-x^2}{h}$

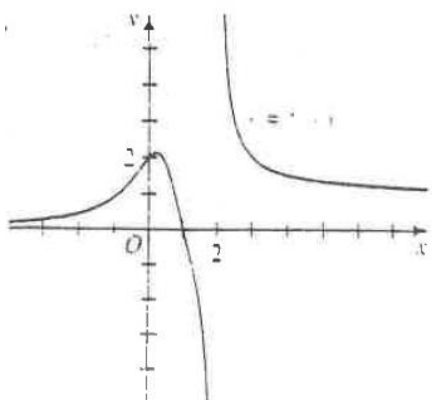
28. a. $\lim_{h\to0} \frac{(1+h)^3-1}{h}$ b. $\lim_{h\to0} \frac{(2+h)^3-8}{h}$ c. $\lim_{h\to0} \frac{(x+h)^3-x^3}{h}$

B17 Side 2

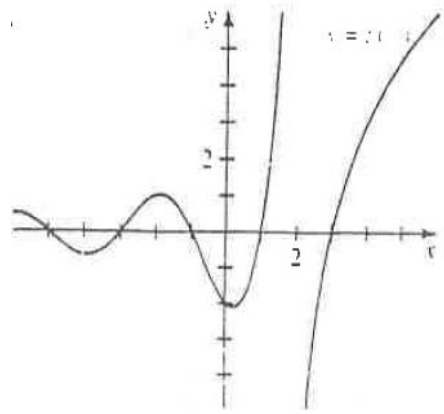
Visual Thinking Use the given function's graph to evaluate each limit

a. $\lim_{x\to\infty} f(x)$ b. $\lim_{x\to-\infty} f(x)$ c. $\lim_{x\to 2^+} f(x)$ d. $\lim_{x\to 2^-} f(x)$

29.



30.



Determine whether each function is continuous. If it is discontinuous, state where any discontinuities occur.

31. $f(x) = \begin{cases} 2 - x^2 & if\ x \le 1 \\ x & if\ x > 1 \end{cases}$

32. $f(x) = \begin{cases} x^2 + 1 & if\ x < 0 \\ x^2 & if\ x \ge 0 \end{cases}$

33. $f(x) = \begin{cases} \frac{x^2-4}{x-2} & if\ x \ne 2 \\ 4 & if\ x = 2 \end{cases}$

34. $f(x) = \frac{x-1}{1-x}$

Evaluate each limit. The result of part (b) of Exercise 43 may be helpful.

44. $\lim_{\theta\to 0} \frac{\tan\theta}{\theta}$
(Hint: $\tan\theta = \frac{\sin\theta}{\cos\theta}$)

45. $\lim_{\theta\to 0} \frac{\sin 2\theta}{\theta}$
(Hint: $\sin 2\theta = 2\sin\theta\cos\theta$)

46. $\lim_{t\to 0} \frac{1-\cos t}{t^2}$
(Hint: Multiply by $\frac{1+\cos t}{1+\cos t}$)

47. $\lim_{x\to\infty} x\sin\frac{1}{x}$
(Hint: Let $t = \frac{1}{x}$. As $x \to \infty, t \to 0$)

Odd Answers

1. ¾ 3. 2 5. ½ 7. 2 9. 5/2 11. ∞ 13. -∞
15. 0 17. 1, -1, does not exist 19. 0, does not exist, does not exist 21. 2
23. -1/2 25. 0 27. 2, 4, 2x 29. 1, 0, ∞, -∞ 31. Cont. 33. Cont.
35. -1, 037. 1 39. 0 41. Does not exist 45. 2 47. 1

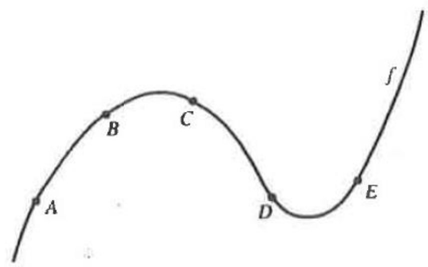
B18 Side 1 Miscellaneous Derivatives

1. Find k so that the given line is tangent to the graph of *f(x)* at *x = 3*

a) $f(x) = x^2 - kx$
$line \quad y = 2x - 9$

b) $f(x) = \frac{k}{x}$
$line \quad y = -\frac{2}{3}x + 4$

2. Using the graph to the right,:



a) Between which two consecutive points is the average rate of change of *f* greatest?
b) Is the average rate of change of *f* between *A* and *B* greater than or less than the instantaneous rate of change at *B*?
c) Sketch the tangent line to the graph between *C* and *D* such that the slope of the tangent line is equal to the average rate of change of the function between *C* and *D*.

3. Find each limit. Note the form for each and how it relates to the limit definition of derivative. This should lead you to a faster way to find the limit for each.

a) $\lim_{h\to 0} \frac{\sqrt{h+2} - \sqrt{2}}{h}$

b) $\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$

c) $\lim_{h\to 0} \frac{\left(2(3+h)^2 + 7\right) - (25)}{h}$

d) $\lim_{x\to 2} \frac{x^3 + x - 10}{x - 2}$

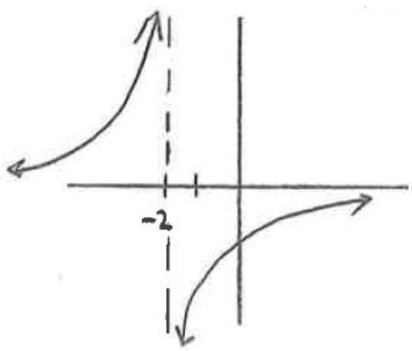
B18 Side 2

4. Determine whether the piecewise function is differentiable

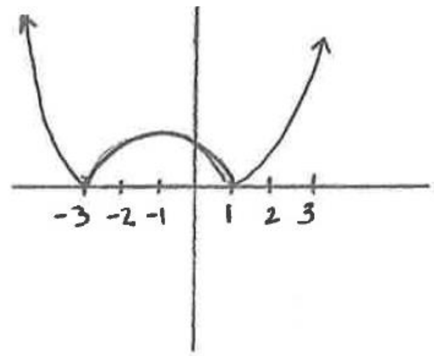
$$f(x)=\begin{cases} x^2+1 & x\le 2 \\ 4x-3 & x>2 \end{cases}$$

5. Determine all values where $f(x)$ is differentiable?

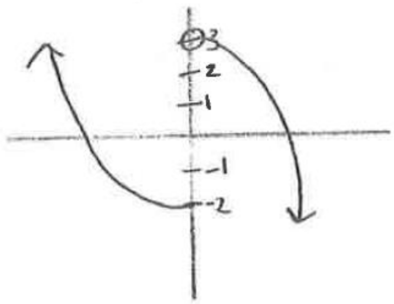
a)



b)



c)



6. Draw a function that satisfies the following:

a) $f(0)=2$
$f'(x)=3$ **for all** x

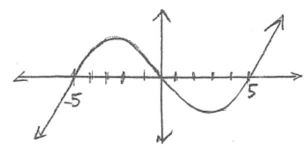
b) $f(0)=4,\ f'(0)=0$
$f'(x)<0 \ \ if \ \ x<0$
$f'(x)>0 \ \ if \ \ x>0$

B19

Graphs of Derivatives Review

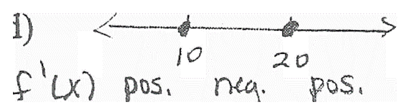
1. Given the following information, draw a sketch of f(x).
   $f(3) = 0,\ f(6) = 0,\ f(0) = 0$
   $f'(0) = f'(2) = 0,\ f'(4) = 0$
   $f''(x) > 0$ *for* $x < 1$ *and for* $x > 3$
   $f''(x) < 0$ *for* $1 < x < 3$

2. Given the graph of $f(x)$ below, answer the following questions:
   a) Where is $f(x)$ increasing?
   b) What does this tell you about $f'(x)$ there?

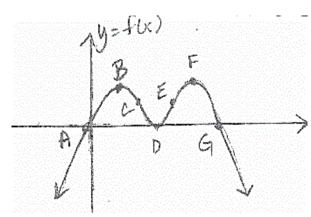


   c) What is $f'(3)$?
   d) At what **approximate** x-value(s) does $f''(x) = 0$?

3. What does each of the following pieces of information tell you about $f(x)$? (Each is a separate question and may refer to different functions)
   a) $f'(x)$ is positive for $3 < x < 8$
   b) $f''(x)$ is negative for $-5 < x < 0$
   c) $f'(3) = 0$
   d) (this is the result of a 1[st] derivative test)



4. Given the graph of $y = f(x)$, graph $y = f'(x)$ and $y = f''(x)$



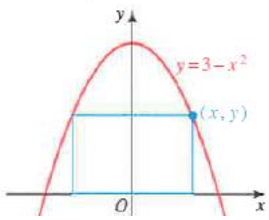
5. Given $f(x) = x^3 + 4x^2 - 11x - 2$ find:
   a) x-intercepts
   b) y-intercepts
   c) local max (for c and d, remember to show use of the first derivative test)
   d) local min
   e) point(s) of inflection
   f) intervals where f(x) is increasing and decreasing
   g) intervals where f(x) is concave up and concave down
   h) sketch

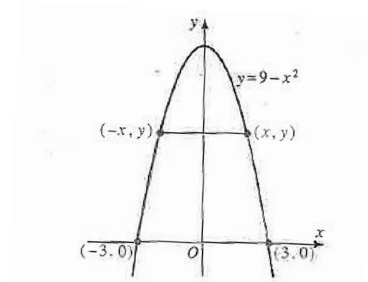
B20 (side 1) Optimization

1. The base of a rectangle is on the x-axis and the upper two vertices are on the parabola $y = 3 - x^2$, as shown at the left below.

   a. show that the area of the rectangle is $A(x) = 6x - 2x^3$, $x \ge 0$

   b. Show that the rectangle has the greatest area when x=1.





2. An isosceles trapezoid is inscribed in the parabola $y = 9 - x^2$ with the base of the trapezoid on the axis, as shown, at the right above.

   a. Show that the area of trapezoid is $A(x) = \frac{1}{2}(9 - x^2)(6 + 2x)$, $x \ge 0$

   b. Show that the trapezoid area has its greatest area when x=1.

3. A rectangle has one vertex at the origin, another on the positive *x*-axis, another on the positive *y*-axis, and the fourth vertex on the line $y = 8 - 2x$. What is the greatest area? What is the greatest area the rectangle can have?

4. A rectangle has area A, where A is constant. The length of the rectangle is x.

   a. Express the width of the rectangle in terms of A and x.

   b. Express the perimeter in terms of A and x.

   c. Find the length and width that minimize the perimeter.

   d. Of all rectangles with a given area, which one has the least perimeter?

5. **Manufacturing** Congruent squares are cut from the corners of a rectangular sheet of metal that is 8 cm wide and 15 cm long. The edges are then turned up to make an open box. Let x be the length of a side of the square cut from each corner.

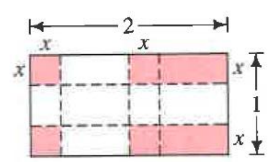
   a. Express the volume V of the box as a function of x.

   b. What value of x maximizes the volume?

6. **Manufacturing** A 1 m by 2 m sheet of metal is cut and folded, as shown, to make a box with a top.

   a. Express the volume V of the box as a function o

   b. What value of x maximizes the volume?



Ex. 6

7. The graph of y=x and $y = x^3$ intersect three times.

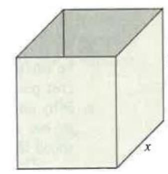
   a. Find the coordinates of the points of intersectic

   b. For what positive value of x between the intersection points is the vertical distance between the graphs the greatest?

B20 side 2

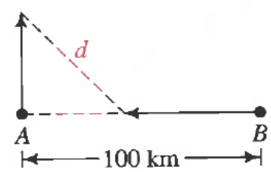
8. **Manufacturing** A manufacturer produces a metal box with a square base, no top, volume of 4000 cm^3.

   a. If the base edges are x cm long, express the height h in terms of *x*.
   b. Show that the area of the metal used in the box is given by the function $A(x) = x^2 + 16000x^{-1}$.
   c. What values of x and h minimize the amount of metal used to make the bc



9. **Business** A toy manufacturer can make x toy tool sets a day at a total cost of $(300 + 12x + 0.2x^{3/2})$ dollars. Each sets sells for 18 dollars. How many sets should be made each day in order to maximize the profit?

10. Point B is 100 km east of point A. At noon, a truck leaves A and travels north at 60 km/h, while a car leaves B and travels west at 80 k/h

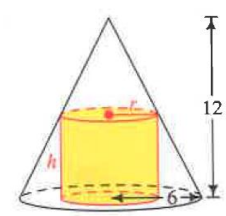


   a. Write an expression for the distance d between the car and the truck after t hours of travel.
   b. When will the distance between the vehicles be a minimum? (Hint: Minimize the square of the distance)
   c. What is the minimum distance between the vehicles?

11. **Manufacturing** A box company produces a box with a square base and no top that has a volume of 8 ft^3. Material for the bottom costs \$6/ ft^2 and the material for the sides costs \$3/ ft^2. Find the dimensions of the box that minimize the cost.

12. A cone has height 12 and radius 6. A cylinder is inscribed in the cone, as sho

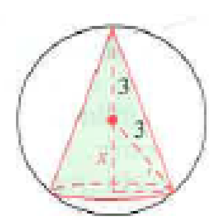
   a. Show that h=12-2r for $0 \le r \le 6$.
   b. Find the maximum volume of the inscribed cylinder.



Ex. 12

13. A cone is inscribed in a sphere with radius 3, as shown at the right.

   a. Show that the volume of the cone is $V(x) = \frac{1}{3}\pi(9 - x^2)(3 + x)$ for $0 \le x \le$
   b. Find the maximum volume of the cone.



Ex. 13

14. Find the maximum volume of cylinder inscribed in a sphere with radius 10.

15. **Business** It costs a construction company (6+0.004x) dollars per mile to operate a truck at x miles per hour. In addition, it costs \$14.40 per hour to pay the driver.

   a. What is the total cost per mile if the truck is driven at 30mi/h? at 40 mi/h?
   b. At what speed should the truck be driven to minimize the total cost per mile?
   c. Why might the company decode to not require the use of the speed found in part (b)?

B21 (side 1) - Optimization

1 Find two real numbers whose difference is 40 and whose product is a minimum.

2 Find two positive real numbers whose sum is 40 and whose product is a maximum.

3 If a box with a square base and open top is to have a volume of 4 cubic feet, find the dimensions that require the least material (neglect the thickness of the material and waste in construction).

4 Work Exercise 3 if the box has a closed top.

5 A fence 8 feet tall stands on level ground and runs parallel to a tall building. If the fence is 1 foot from the building, find the shortest ladder that will extend from the ground over the fence to the wall of the building.

6 A page of a book is to have an area of 90 square inches, with 1-inch margins at the bottom and sides with a ½-inch margin at the top. Find the dimensions of the page which will allow the largest printed area.

7 Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius *a,* if two vertices lie on the diameter.

8 Find the dimensions of the rectangle of maximum area that can be inscribed in an equilateral triangle of side *a*, if two vertices of the rectangle lie on one of the sides of the triangle.

9 Of all possible right circular cones that can be inscribed in a sphere of radius *a*, find the volume of the one which has maximum volume.

10 Find the dimensions of the right circular cylinder of maximum volume that can be inscribed in a sphere of radius *a*.

11 A metal cylindrical container with an open top is to hold one cubic foot. If there is no waste in construction, find the dimensions which require the least amount of material.

12 If the circular base of the container in Exercise 11 is cut from a square sheet and the remaining metal is discarded, find the dimensions which require the least amount of material.

13 A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides at right angles to the sheet. How many inches should be turned up to give the gutter its greatest capacity? (ℓ is constant)

14 Work Exercise 13 if the sides of the gutter make an angle of 120 with the base.

15 Prove that the rectangle of largest area having a given perimeter *p* is a square.

16 A right circular cylinder is generated by rotating a rectangle of perimeter *p* about one of its sides. What dimensions of the rectangle will generate the cylinder of maximum volume

17 The strength of a rectangular beam varies jointly as the width and the square of the depth of a cross section. Find the dimensions of the strongest beam that can be cut from a cylindrical log of radius *a*.

18 A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions which will allow the maximum amount of light to enter.

B21 – side 2

19 Find the point on the graph of $y = x^2 + 1$ that is closest to the point (3, 1).

20 Find the abscissa of the point on the graph $y = x^3$ that is closest to the point (4,0).

21 A manufacturer sells a certain article to dealers at a rate of $20 each if less than 50 are ordered. If 50 or more are ordered (up to 600), the price per article is reduced at a rate of 2 cents times the number ordered. What size order will produce the maximum amount of money for the manufacturer?

22 Refer to Example 5 of this section. If the man is in a motorboat which can travel at an average rate of 15 miles per hour, how should he proceed in order to arrive at his destination in the least time?

23 The illumination from a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources of strengths S_1 and S_2 are d units apart, at what point on the line segment joining the two sources is the illumination minimal?

24 At 1:00 P.M, ship A is 30 miles due south of ship B and is sailing north at a rate of 15 miles per hour. If ship B is sailing west at a rate of 10 miles per hour, at what time will the distance between the ships be minimal?

25 A veterinarian has 100 feet of fencing and wishes to construct six dog kennels by first building a fence around a rectangular region, and then subdividing that region into six smaller rectangles by placing five fences parallel to one of the sides. What dimensions of the region will maximize the total area?

26 A paper cup having the shape of a right circular cone is to be constructed. If the volume desired is 36π in.^{3}, find the dimensions that require the least amount of paper (neglect any waste that may occur in construction).

27 A steel storage tank for propane gas is to be constructed in the shape of a right circular cylinder with a hemisphere at each end. If the desired capacity is 100 ft^3, what dimensions will require the least amount of steel?

28 A pipeline for transporting oil will connect two points *A* and *B* which are 3 miles apart and on opposite bank of a straight river one mile wide. Part of the pipeline will run under water from *A* to point *C* on the opposite bank, and then above ground from *C* to *B*. If the cost per mile of running the pipeline under water is four times the cost per mile of running it above ground, find the location of *C* which will minimize the cost (ignore the slope of the river bed).

29 A wire 36 cm long is to be cut into two piece. One of the pieces will be bent into the shape of an equilateral triangle and the other into the shape of a rectangle whose length is twice its width. Where should the wire be cut if the combined area of the triangle and rectangle is (a) a minimum? (b) a maximum?

30 An isosceles triangle has base *b* and equal sides of length *a*. Find the dimensions of the rectangle of maximum area that can be inscribed in the triangle if one side of the rectangle lies on the base of the triangle.

B22 (side 1) - Velocity and Acceleration

1. **Physics** A ball is thrown upward from the top of an 80 ft building so that its height in feet above the ground after t seconds is $h(t) = 80 + 64t - 16t^2$.

   a. Find the average velocity over the interval from t = 0 s to t = 2 s.
   b. What is the instantaneous velocity at time t? At time t = 1 s?
   c. When is the velocity equal to 0?
   d. What is the ball's maximum height above the ground?
   e. When does the ball hit the ground?
   f. For what values of t is the ball failing?
   g. What is the acceleration at time t?
   h. Graph the function h(t).

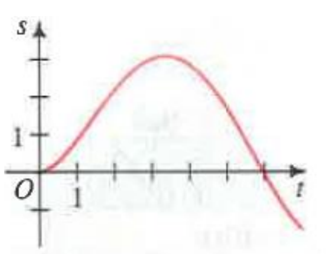
2. **Physics** A helicopter climbs vertically from the top of a 98 m building so that its height in meters above the ground after *t* seconds is: $h(t) = 98 + 49t - 4.9t^2$

   a. Find the average velocity over the interval from *t* = 0 s to *t* = 2 s.
   b. What is the instantaneous velocity at time *t*? At time *t* = 1 s?
   c. When is the velocity equal to 0?
   d. What is the helicopter's maximum height above the ground?
   e. When does the helicopter reach the ground?
   f. For what values of *t* is the helicopter descending?
   g. What is the acceleration at time *t*?
   h. Graph the function *h*(*t*).

Visual Thinking In Exercises 3 and 4, answer the questions: In each graph, *s* is the distance (in meters) to the right of the starting point and *t* is the time (in seconds).

   a. How far does the toy car travel in the first two seconds?
   b. What is its average velocity during the first two seconds?
   c. When does the car move to the right? To the left?
   d. When does the car return to its starting point?
   e. What is the instantaneous velocity of the car at t = 1 s? t = 2 s? t = 3 s?
   f. At what times does the car move fastest to the right? To the left?
   g. What is the farthest the car moves to the right during the first 6 seconds?

3.



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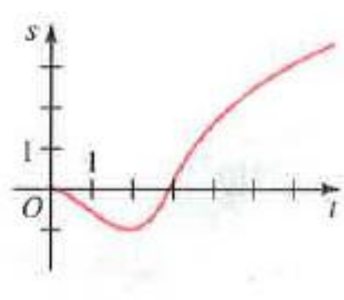


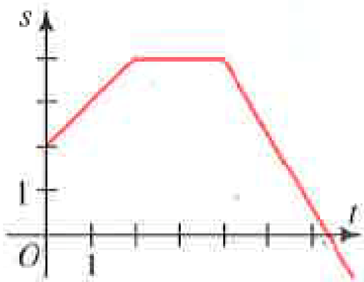
B22 – side 2

5. **Visual Thinking** Study the graph of $s(t)$ in Exercise 3. The sketch an approximate graph of the velocity function $v(t)$.

6. **Visual Thinking** Study the graph of $s(t)$ in Exercise 4. Then sketch an approximate graph of the velocity function $v(t)$.

Visual Thinking In Exercises 7 and 8, copy the given graph of $s(t)$. Then, on separate sets of axes aligned vertically with the copied graph, sketch the graphs of $v(t)$ and $a(t)$.

7.



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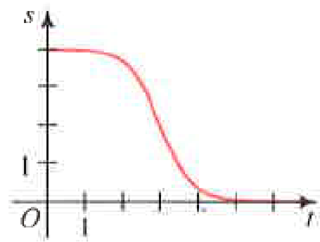


Visual Thinking In Exercises 9 and 10, read the given description of the motion of a particle along a number line. Then make an approximate graph (like those in Exercises 3 and 4) showing the distance of the particle to the right of the origin as a function of time.

9. A particle leaves the origin, traveling at a velocity of 1m/s. Then it gradually slows until it reverses its direction at time t = 3*s*. It returns to the origin at t = 5s, traveling at a velocity of -2 m/s and then gradually slows until it stops to the left of the origin at time t = 10*s*.

10. A particle leaves the origin, traveling right very slowly, and gradually increasing its velocity until it reaches 2m/s at time t = 4*s*. Then the velocity begins to decrease and finally becomes zero at t = 8*s*. At this time, the particle begins to move slowly left, and at t = 20*s* it finally returns to its starting point and stops.

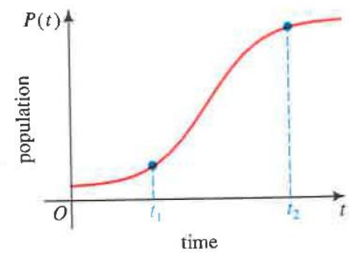
11. **Economics** If $P(t)$ represents the general price level of goods and services in an economy at a given time t, then during an inflationary period, $P(t)$ increases rapidly. Suppose that our economy is in such an inflationary period, but that the president's economic advisers find that the rate of inflation is slowing. In a subsequent address to the nation, the president announces that "while inflation is still with us, it has been brought under control" and predicts that "prices will soon stabilize."

a. Use a derivative to describe why "inflation is still with us."

b. Use a derivative to describe why the president believes that inflation "has been brought under control."

c. Use a derivative to describe the president's prediction that "prices will soon stabilize."

B22 – side 3



12. **Biology** At the right is the graph of $P(t)$, which shows the growth over time in the population of a protected endangered species.
a. Compare the signs of $P'(t_1)$ and $P''(t_1)$ with $P'(t_2)$ and $P''(t_2)$ at times t_1 and t_2.
b. *Writing* What conclusions might you expect biologists who are studying the endangered species to make at times t_1 and t_2?

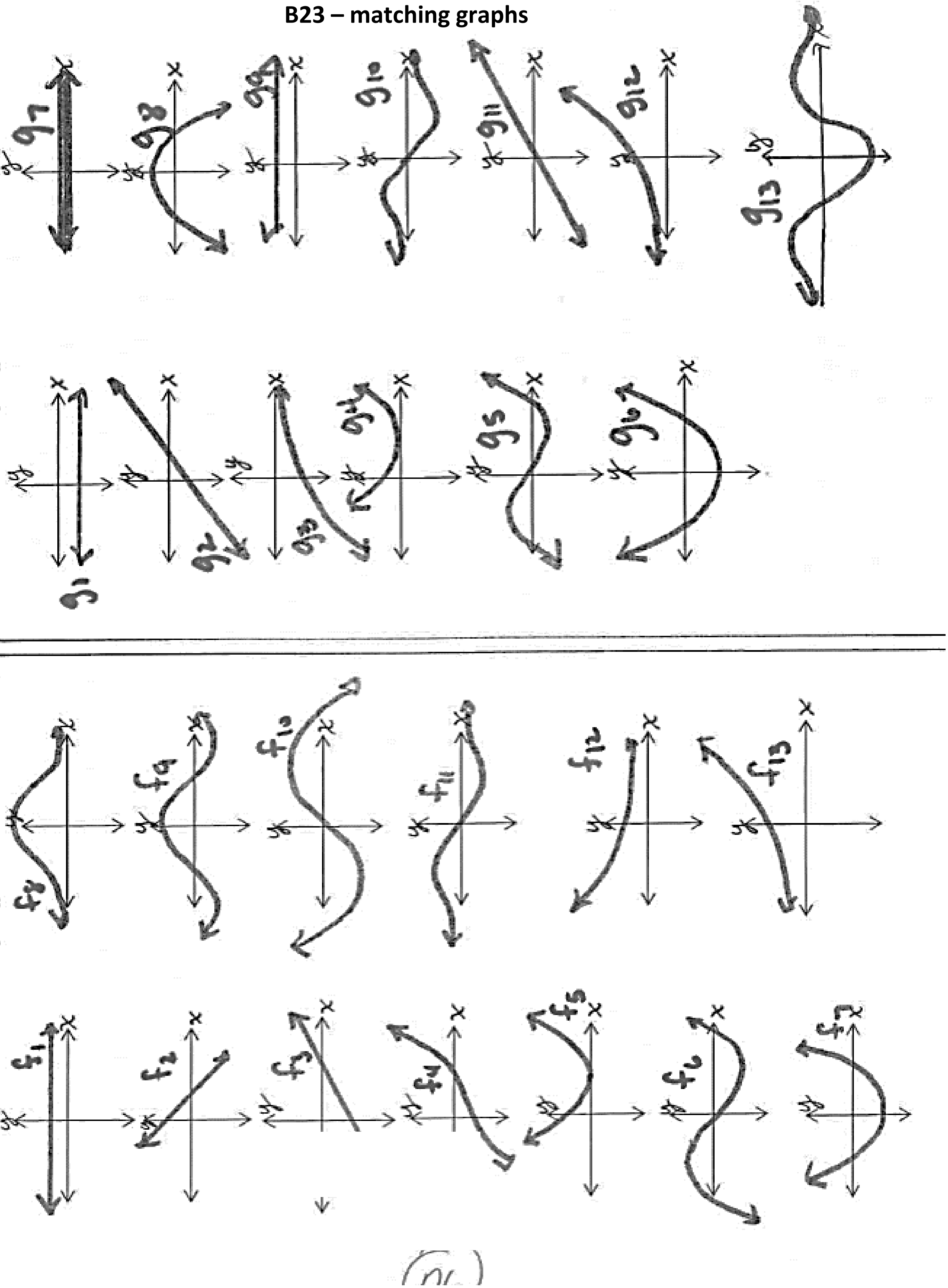
13. **Physics** A particle moves along a number line so that its distance to the right of the origin at time t is $s(t) = 2t^3 - 6t^2 + 8$.
a. At what times is the particle at the origin?
b. At what times is the particle not moving?
c. At what times is the velocity of the particle neither increasing nor decreasing; that is, at what time is the particle neither accelerating nor decelerating?

14. **Physics** Suppose a car is traveling so that its distance, in miles, west of Rockford at a time t hours after noon is $s(t) = 10t^{3/2} - 15t + 20$.
a. How fast is the car going, and in which direction, at 12:15 P.M?
b. Where is the car, and what is the time, when its velocity is zero?

15. **Physics** A ball is thrown horizontally from the top of a 144 ft. building with an initial speed of 80 ft./s. Unlike Exercise 1 where the motion of the ball is along a line, here the motion is in a plane and the position (x, y) of the ball at time t seconds is a vector $s(t) = (x(t), y(t)) = (80t, 144 - 16t^2)$.
a. Make a sketch showing the path of the ball and its position at times t = 0s, t = 1s, t = 2s, and t = 3s.
b. Show that the distance between the ball's positions at times t = 1s and t = 2s is approximately 93.3 ft.
c. From t = 1s to t = 2s, the ball actually traveled along a curved path which is a little longer than 93.3 ft. Thus, its average speed over the time interval is somewhat more than 93.3 ft./s. Use similar reasoning to find an approximate average speed of the ball over the time interval from t = 1.9 s to t = 2.0 s.
d. Using calculus, one can show that the instantaneous velocity of the ball is the vector $v(t) = \left(x'(t), y'(t)\right)$. In other words, the components of the velocity vector are the derivatives of the components of the position vector. Moreover, the speed of the ball at time t is $|\vec{v}(t)| = \sqrt{(x'(t))^2 + (y'(t))^2}$. Use this information to find the velocity vector and speed of the ball when t = 2 sec and at the instant the ball hits the ground.

B23 – matching graphs

For each of the graphs of the functions $f_1 - f_{13}$, match the graph of the function's derivative on the right $(g_1 - g_{13})$.

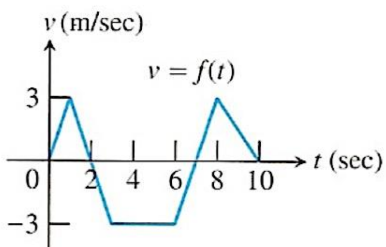


B24 – Rectilinear motion practice

1. You may use a graphing calculator to solve parts of this problem. A particle moves along a horizontal line so that its position at time $t \geq 0$ is given by $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters and *t* is in seconds.

a) Find the average velocity in the first two seconds.
b) Find the instantaneous velocity when *t* = 2.
c) Find the instantaneous acceleration when *t* = 2.
d) When is the particle at rest? When is it moving to the right? To the left? Support your answer.
e) Find the displacement of the particle during the first 2 seconds.
f) Find the total distance traveled by the particle during the first two seconds.
g) Are the answers to e) and f) the same? Explain.
h) When is the particle speeding up? Slowing down? Justify your answer.

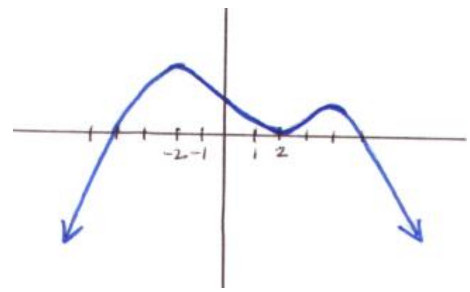
2. The figure below shows the velocity of a particle moving along a horizontal line.



a) When is the particle moving left? Right? At rest? Support your answers.
b) When is the particle moving at a constant speed?
c) Graph the particle's speed.
d) Graph the particle's acceleration.
e) When is the particle speeding up? Slowing down?

B25 – Interpreting graphs

1. Given the graph of $y = f'(x)$, the derivative of $y = f(x)$, determine the following about $y = f(x)$:



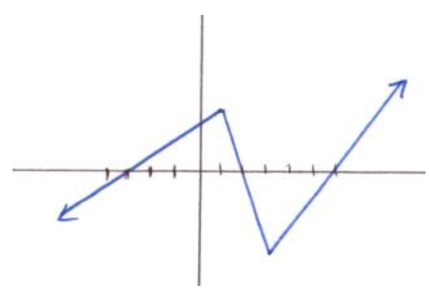
a) intervals where $y = f(x)$is increasing/decreasing.

b) x-coordinate of relative maximum/minimum

c) intervals of concavity

d) x-coordinate of points of inflection

2. Given the graph of $y = f'(x)$, the derivative of $y = f(x)$, determine the following about $y = f(x)$:



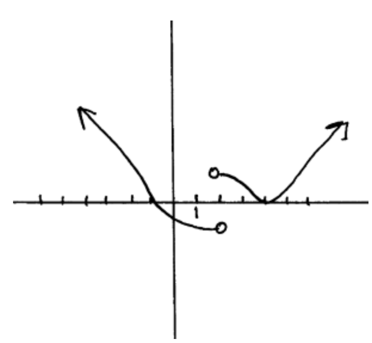
a) intervals where $y = f(x)$is increasing/decreasing.

b) x-coordinate of relative maximum/minimum

c) intervals of concavity

d) x-coordinate of points of inflection

3. Given the graph of $y = f'(x)$, the derivative of $y = f(x)$, determine the x-coordinate of relative maximum/minimum



B26 – Area under a Curve

For problems 1-5; find the area under the curve over the given interval using the given number of rectangles.

1. $y = x^2 + 2x$, [0, 2]

a) RRAM using 6 rectangles b) LRAM using 6 rectangles c) MRAM using 3 rectangles

2. $y = \sqrt{x} + 2$, [1, 2]

a) RRAM using 4 rectangles b) LRAM using 4 rectangles c) MRAM using 2 rectangles

3. $y = x^3 + 2x^2 + x + 1$, [-1, 1]

a) RRAM using 6 rectangles b) LRAM using 6 rectangles c) MRAM using 3 rectangles

4. $y = 3x^2$, [-1, 1]

a) RRAM using 8 rectangles b) LRAM using 8 rectangles c) MRAM using 4 rectangles

5. $y = \sqrt{4 - x^2}$, [-2, 2]

a) RRAM using 8 rectangles b) LRAM using 8 rectangles c) MRAM using 4 rectangles

For problems 6-9, find the area under the curve using RRAM and the given number of subintervals, then find the sum of n subintervals

6. $y = 2x^3 + 1$, [0, 2], 4 subintervals

7. $y = x^3 - 2x^2 + 5$, [-1, 3], 12 subintervals

8. $y = 2x^2 + x - 3$, [1, 3], 5 subintervals

9. $y = -x^2 + 4$, [-2, 2], 8 subintervals

B27 – Areas

1. a) Approximate the area under the graph of $f(x) = \frac{1}{x}$ from *x* = 1 to *x* = 5 using the **right endpoints** for four subintervals of equal length. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

b) Repeat part a) using **left endpoints**.

2. Approximate the area under the graph of $f(x) = 25 - x^2$ from *x* = 0 to *x* = 5 using the **midpoints** of five subintervals of equal length. Sketch the graph and the rectangles.

3.

x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

a) Given the table of values above, approximate the area under the curve $y = f(x)$ from *x* = -5 to *x* = 5 using the **left endpoints** for four subintervals.

b) Repeat part a) above using **right endpoints**.

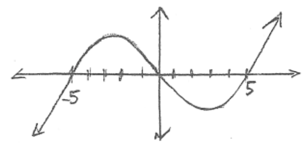
c) Why can you not do this problem with midpoints of four subintervals?

d) Approximate the area under the curve using the midpoints of 2 subintervals.

B28 - Applications of Derivatives Review

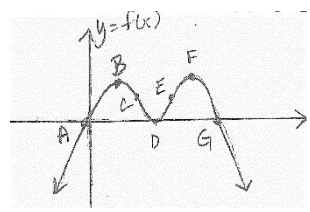
1. Given the following information, draw a sketch of f(x).
   $f(3) = 0$, $f(6) = 0$, $f(0) = 0$
   $f'(0)$, $f'(2) = 0$, $f'(4) = 0$
   $f''(x) > 0$ *for* $x < 1$ *and for* $x > 3$
   $f''(x) < 0$ *for* $1 < x < 3$

2. Given the graph of $f(x)$ below, answer the following questions:
   a) Where is $f(x)$ increasing?
   b) What does this tell you about $f'(x)$ there?



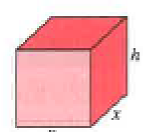
   c) What is $f'(3)$?
   d) At what **approximate** x-value(s) does $f''(x) = 0$?

3 Given the graph of $y = f(x)$, graph $y = f'(x)$ and $y = f''(x)$



4. An object moves so that its distance to the right of the origin is described by $s(t) = 2x^3 – 7x^2 + 6x – 1$. Find:
   a) Average velocity from $t = 1$ sec to $t = 8$ sec
   b) Instantaneous velocity at $t = 6$ sec
   c) Average acceleration from $t = 1$ sec to $t = 4$ sec
   d) Instantaneous acceleration at $t = 3$ sec
   e) For what values of t is the object left of the origin?

5. a. A rectangular solid has a square base, as shown. Express its surface area A as a function of x and h.
   b. If the volume of the solid is 1000 cubic units, express h in terms of x.
   c. Use parts (a) and (b) to express the surface area A as a function of x alone.
   d. Show that the surface area is minimized when $x = h$, that is, when the solid is a cube.



6. **Physics** A particle is moving along a number line so that its position to the right of the origin in time t is $s(t) = 3t^2 – 5t + 4$. Find the position, velocity, and acceleration of the particle at time $t = 5$.

7. **Physics** An arrow is shot upward from the bottom of a canyon that is 336 ft. below the edge of a cliff. If the initial velocity of the arrow is 160 ft./s, and $s(t) = 160t – 16t^2$ gives the arrow's height, in feet, above the bottom of the canyon t seconds after the arrow is shot.
   a. In how many seconds does the arrow pass the edge of the cliff on its upward trip?
   b. How high does the arrow travel before it starts to descend?
   c. If it lands on the cliff's edge, how long was the arrow in the air?
   d. At what velocity does the arrow hit the cliff's edge?

ANSWERS

B1: Limits

1. -1, 3, DNE, 1, -1, 3
2. 2, 0, DNE, 2, 0, 2
3. 1, 1, 1, 1, $-\infty$, ∞
4. 3, 3, 3, 3, ∞, ∞
5. 0, 0, 0, 3, ∞, ∞
6. 2, 2, 2, 3, $-\infty$, ∞
7. $-\infty$, ∞, DNE, undefined, 2, 0
8. $-\infty$, $-\infty$, $-\infty$, 1, 2, 2
9. 1, $-\infty$, DNE, -2, ∞, ∞
10. 3, 3, 3, 3, DNE, 0
11. ($-\infty$, -4) U (-4, ∞)
12. ($-\infty$, 3) U (3, ∞)

B2: Trig Limits

1. 1 2. 2 3. 1 4. $\frac{5}{3}$ 5. 0 6. 0 7. $\frac{4}{3}$ 8. $\frac{1}{2}$ 9. 1 10. -1

B3:

A:

1. 13 2. 20 3. $-4-\sqrt{14}$ 4. 4 5. $\frac{7}{8}$ 6. $\frac{11}{3}$ 7. $\frac{32}{3}$ 8. ∞

9. ∞ 10. $-\frac{1}{25}$ 11. 3 12. 5 13. -1 14. $\frac{\sqrt{2}}{4}$ 15. $4a^3$ 16. $\frac{1}{3}$

17. $\frac{-3}{16}$ 18. 1 19. -1 20. DNE

B:

1. 4 2. 3 3. -6 4. $-\frac{25}{4}$ 5. 3 6. 0 7. -1 8. 1

9. 1 10. -1 11. $\frac{1}{8}$ 12. 0 13. 1 14. -1 15. ∞ 16. $-\infty$

17. 6, 4 18. Ask in class

B4: continutiy

1. Not continuous 17. Not continuous 19. 1 21. Continuous 23. Not continuous 39. a=2

40. a=-1, b=1 41. 2. -1 43. Cont. on $(-\infty,\infty)$ 44. Cont. on $[-3,\infty)$

B5 (top): Difference Quotient

1. 4x+2h-3
2. $3x^2+3xh+2+h^2$
3. $-\frac{1}{(x+h-4)(x-4)}$
4. $\frac{-2x-h-4}{(x+2)^2(x+h+2)^2}$
5. $\frac{1}{(x+1)(x+h+1)}$
6. $\frac{3}{\sqrt{3x+3h-4}+\sqrt{3x-4}}$

B5 (bottom): Intermediate Value Theorem

1. f(x) is continuous because it is a polynomial.

   f(-3)=16 f(2)=1 and f(2)=1<4<16=f(-3) so, the Intermediate Value Theorem applies
   Therefore there exists a $c\epsilon[-3,2]$ such that $f(c) = 4$
   c = -1 (you have to show the work to get to this point.

2. $c = 2$
3. $c = \frac{1}{2}$
4. $c = 1$
5. $c = \frac{3}{2}$
6. $c = 2$

B6:

1. a) -3 b) $-\infty$ 3 a) -12 b) 21 c) -15 d) 25 e) 2 f) $-\frac{3}{5}$ g) 0 h) undefined 5. ∞ 7. $-\infty$ 9. ∞

11. $\frac{3}{2}$ 13. 0 15. 0 16. $\frac{5}{3}$ 17. $-\frac{\sqrt[3]{5}}{2}$ 18. $\frac{\sqrt[3]{12}}{2}$ 19. $-\sqrt{5}$ 21. $\frac{1}{\sqrt{6}}$ 23. $\sqrt{3}$ 25. $-\infty$ 27. $-\frac{1}{7}$

29. a) ∞ b) -5 31. 0 32. $-\frac{3}{2}$ 33. $\frac{a}{2}$

B7: Definition of Derivative

1. 2x 2. $\frac{-1}{2x\sqrt{x}}$ 3. $\frac{-1}{2\sqrt{1-x}}$ 4. $\frac{-2x}{\left(x^2+1\right)^2}$ 5. 4 6. 0

7. y-4=3(x-1) 8. $y-3=\frac{1}{6}(x-9)$

B8: Differentiability

8. $y-9=\frac{1}{2\sqrt{3}}(x-3)$ 9. $x=0$ 10. $x=2$ 11. $x=\pm 3$ 12. $x=0,\ x=4$ 13. Yes; 0

14. no 15. No 16. Yes; 1

B9:

1. $\frac{2}{5}$	2. 2	3. 3	4. ∞	5. 0	6. ∞	7. -1	8. 0
9. 0	10. $-\frac{1}{2}$	11. $\frac{5}{9}$	12. 5	13. $\frac{3}{2}$	14. $-\infty$	15. ∞	
	16. $\frac{1}{2}$						
17. $-\frac{1}{2}$	18. $\frac{1}{2}$	19. -7	20. $\frac{1}{4}$	21. $-\frac{1}{2}$	22. $\frac{5}{3}$	23. $-\frac{1}{7}$	24. 0
25. 0	26. 12	27. 1	28. 0	29. 1	30. 0	31. 0	32. 0
33. DNE	34. 0	35. 2	36. 8	37. 1	38. 1	39. 1	
	40. 2						

41. 0 42. 1 43. 1 44. DNE 45. 6 46. 0 47. -9 48. -4

B10:

6. $\frac{6}{\sqrt[4]{x}}$ 7. $-\frac{6}{x^3}$ 8. $-\frac{2}{x\sqrt{x}}$ 10. $-\frac{1}{x^5}$ 11. $7x-5+\frac{1}{x^2}$ 16. $\frac{1}{3}$ 17. -13

19. $\frac{1}{2}$ 21.a) $f'(x)=2ax+b$ b) in class 25. $y-1=2(x+1)$ 27.a) insert graph b) x = 0, x = 4

31. $f(x)=x^4+c$ 33. $g(x)=\frac{1}{2}x^6+c$

B11 Sketching polynomials

B11 answers

(3) Show 1st deriv test

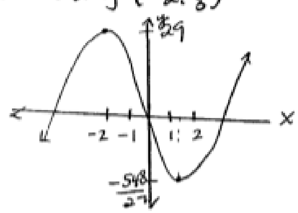
f': + | − | + (critical points at −2 and $\frac{5}{3}$)

local max = 29 at x = −2

local min = $-\frac{548}{27}$ at x = $\frac{5}{3}$

increasing $(-\infty,-2)\cup(\frac{5}{3},\infty)$

decreasing $(-2,\frac{5}{3})$



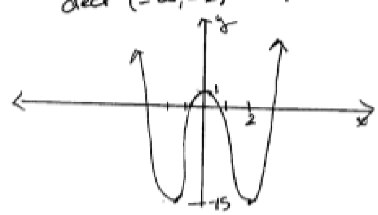
(5) f': − | + | − | + (critical points at −2, 0, 2)

local max = 1 at x = 0

local min = −15 at x = ±2

incr $(-2,0)\cup(2,\infty)$

decr $(-\infty,-2)\cup(0,2)$

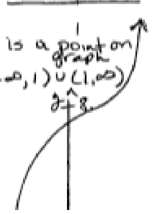


(6) f': + | + (at 1)

no local extrema but slope of tangent at x = 0 is 0

(1,8) is a point on graph

incr $(-\infty,1)\cup(1,\infty)$



(12) f': + | + | − (at 0, 1)

F41: (0,0) is not an extremum. It is a point of inflection.

local max = 1 at x = 1

incr $(-\infty,0)\cup(0,1)$

decr $(1,\infty)$

(23) a) abs min = −6 at x = 1

abs max = $\frac{129}{16}$ at x = $-\frac{7}{8}$

Be sure to use the Candidates test

b) abs min = −31 at x = −4

abs max = $\frac{129}{16}$ at x = $-\frac{7}{8}$

c) abs min = −130 at x = 5

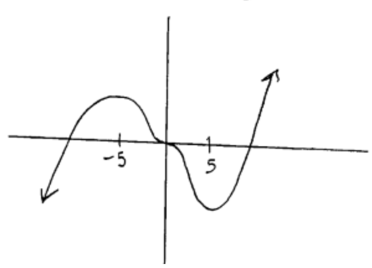
abs max = 5 at x = 0

(25) b) abs max = 29 at x = −2

abs min = −31 at x = −4

$y=x^3$

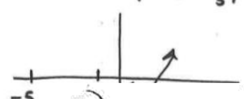
(27) f': + | − | − | + (at −5, 0, 5)



B12 Concavity

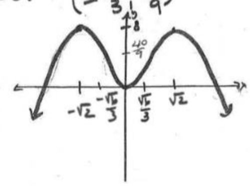
① local max = $\frac{31}{27}$ at $x = \frac{1}{3}$
local min = 1 at $x = 1$
p.o.i. = $\left(\frac{2}{3}, \frac{29}{27}\right)$
f(x) is concave: up $\left(\frac{2}{3}, \infty\right)$
down $\left(-\infty, \frac{2}{3}\right)$

② local min = $-\frac{1850}{27}$ at $x = -\frac{5}{3}$
local max = -50 at $x = -5$
Concave down $\left(-\infty, -\frac{10}{3}\right)$
Concave up $\left(-\frac{10}{3}, \infty\right)$



③ local max – none
local min = 5 at $x = 1$
p.o.i. $\left(\frac{2}{3}, \frac{146}{27}\right)$ $5\frac{11}{27}$
(0, 16)
f(x) is concave:
up $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$
down $\left(0, \frac{2}{3}\right)$

④ f′: incr (+) $-\sqrt{2}$ decr (−) 0 incr (+) $\sqrt{2}$ decr (−)
local max, local min, local max
local max = 8 at $x = \pm\sqrt{2}$
local min = 0 at $x = 0$
f(x): c.d. poi c.u. poi c.d.
f″(x): (−) $-\frac{\sqrt{6}}{3}$ (+) $\frac{\sqrt{6}}{3}$ (−)
poi: $\left(\pm\frac{\sqrt{6}}{3}, \frac{40}{9}\right)$

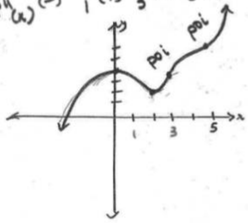


⑦ local max = 1 at $x = 0$
local min: 0 at $x = 1$
0 at $x = -1$
poi $\left(-\frac{\sqrt{3}}{3}, \frac{4}{9}\right)$ $\left(\frac{\sqrt{3}}{3}, \frac{4}{9}\right)$
f(x) is Concave:
up $\left(-\infty, -\frac{\sqrt{3}}{3}\right) \cup \left(\frac{\sqrt{3}}{3}, \infty\right)$
down $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

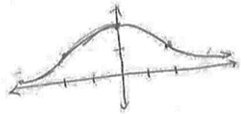
⑧ no local extrema
$f''(x) = -\frac{32}{x^3}$ never = 0
f: cu, cd
f″: + 0 −
f(x) is concave:
up $(-\infty, 0)$
down $(0, \infty)$
eba $y = x$

⑲ points (0,1) (2,3)
CP (0,1) (2,3)
f(x): decr, local min, incr, local max, decr
f′(x): − 0 + 2 −
f(x): c.u. poi c.d.
f″(x): + 1 −

⑳ pts (0,4) max
(2,2) min
(5,6) poi
f(x): incr, local max, decr, local min, incr
f′(x): (+) 0 (−) 2 (+)
f(x): c.d. poi c.u. poi c.d. poi c.u.
f″(x): (−) 1 (+) 3 (−) 5 (+)



㉑ points (0,2)
(2,1)
(−2,1)
f(x): incr, local max, decr
f′(x): + 0 −
cu poi cd poi cu
+ −2 − 2 +
(2,1) (−2,1) are points of infl



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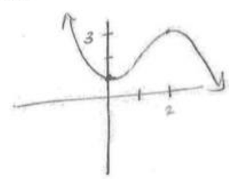
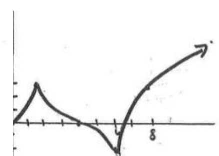




f(x): incr, local max, decr, local min, incr
f′(x): (+) 2 (−) 6 (+)

f(x): c.d. poi c.u. c.u. poi c.d. c.d.
f″(x): − 0 + 2 + 4 − 6 −

#27, 29 discuss in class

B13-Justifying answers

1a) Because $f'(x)$ changes sign from negative to positive at $x=\frac{3}{2}$, f has a local minimum = $-27/8$ at $x=\frac{3}{2}$ by the First Derivative Test.

b) Because $f'(\frac{3}{2})=0$ and $f''(\frac{3}{2})>0$, f has a local minimum = $-27/8$ at $x=\frac{3}{2}$ by the Second Derivative Test

2a) Because $f'(x)$ changes sign from negative to positive at $x=-\sqrt{2}$, f has a local minimum = -8 at $x=-\sqrt{2}$ by the First Derivative Test.

Because $f'(x)$ changes sign from positive to negative at $x=0$, f has a local maximum = 0 at $x=0$ by the First Derivative Test.

Because $f'(x)$ changes sign from negative to positive at $x=\sqrt{2}$, f has a local minimum = -8 at $x=\sqrt{2}$ by the First Derivative Test.

b) Because $f'(-\sqrt{2})=0$ and $f''(-\sqrt{2})>0$ f has a local minimum = -8 at $x=-\sqrt{2}$ by the Second Derivative Test.

Because $f'(0)=0$ and $f''(-\sqrt{2})=0$ the Second Derivative Test fails so we cannot use it to determine max/min.

Because $f'(\sqrt{2})=0$ and $f''(\sqrt{2})>0$ f has a local minimum = -8 at $x=\sqrt{2}$ by the Second Derivative Test.

3. $f(-2)=32$ absolute minimum = 32 at x = -2
$f(0)=0$ absolute maximum = 0 at x = 0.
$f(1)=-12$

B14 - Product/Quotient Rules

6. $4x-1$
21. $5x^4+3x^2+2x$
22. $\frac{1}{2\sqrt{x}}-3$
23. $14x^6-4x^3-6$
25. $\frac{9}{y^4}+\frac{14}{y^2}+5$
26. $y'=\frac{3}{2}\sqrt{x}-\frac{1}{2\sqrt{x}}$
27. $g'(x)=\frac{5}{(2x+1)^2}$
28. $f'(t)=\frac{-2t^2+8}{(4+t^2)^2}$
30. $y'=\frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$
32. $y'=\frac{-t-1}{(t-1)^3}$
34. $g'(t)=-\frac{1}{3t\sqrt[3]{t}}+\frac{5}{6t\sqrt[6]{t^5}}$
36. $y'=\frac{-B}{x^2}-\frac{2C}{x^3}$
38. $y'=\frac{C}{(1+Cx)^2}$
40. $y'=4u^3-2-\frac{10}{u^3}$
42. $f'(x)=\frac{ad-bc}{(cx+d)^2}$
49. $y-1=\frac{1}{2}(x-1)$
55. $y-2=-\frac{1}{2}(x-1)$; $y-2=2(x-1)$

64.a) $h'(2)=-38$ b) -29 c) $\frac{13}{16}$ d) $-\frac{3}{2}$

65. 16
66. $-\frac{5}{2}$
68.a) $\frac{3}{2}$ b) $\frac{43}{12}$

69.a) $y' = xg'(x) + g(x)$ b) $y' = \frac{g(x) - xg'(x)}{(g(x))^2}$ c) $y' = \frac{xg'(x) - g(x)}{x^2}$

71. $(-2, 21), (1, -6)$ 72. $x = \frac{-3 \pm \sqrt{6}}{3}$ 74. $y - 8 = 3(x - 4)$ 76. $y = \frac{1}{2}(x-1)$; $y - 2 = \frac{1}{2}(x+3)$

B15: Review Limits/Functions

1. 1   2. 4   3. $\frac{1}{2}$ 4. $\frac{15}{2}$ 5. 13   6. 3   7. -1   8. $\frac{1}{2\sqrt{3}}$

9. $-\frac{1}{25}$ 10. $-\frac{25}{4}$ 11. $\frac{1}{\sqrt{3}}$ 12. $-\infty$ 13. 3, 1, DNE

14.a) D:[-2, 3]   x intercepts: (-2, 0) (3, 0)   check on gc   b) D: $(-\infty, -3)U(-3, 3)U(3, \infty)$
x intercept: (0, 0)   y intercept: (0, 0)   hole (3, 3)   va x = 3   eba y=2x-6

c)D: $(-\infty, -4)U(-4, 2)U(2, \infty)$ x intercept: (-2, 0)   y intercept: $(0, -\frac{1}{4})$ va x = -4   eba: y = 0

15.a)not continuous at x = 1   b) continuous at x = 8   16k = 1, k = 2

B16: Review of Derivatives

1.a) $\frac{3}{2\sqrt{3x-1}}$ b) 2x-3   2. $f'(x) = -\frac{2}{81}$ 3. $y - 22 = 25(x-2)$ 4. $y = \frac{3}{2}(x-1)$

5.a) $(\frac{1}{3}, \frac{23}{3})$ b) $(\pm\frac{\sqrt{3}}{3}, \frac{2}{3})$, $(0,1)$ 6.a) Yes at x = 0.   b) no vertical tangent

7.a) $\frac{2}{\sqrt[3]{x}} - 15x^2$ b) $\frac{1}{2\sqrt{x}} - 8x - \frac{2}{x^2}$ c)0

8. $f'(x) = \frac{20}{3}x^{\frac{1}{3}} - 21x^2 - \frac{8}{x^3} + 5$; $f''(x) = \frac{20}{9x^{\frac{2}{3}}} - 42x + \frac{24}{x^4}$; $f'''(x) = \frac{-40}{27x^{\frac{5}{3}}} - 42 - \frac{96}{x^5}$

B17: odd answers on the bottom of worksheet

Odd answers:

1. ¾	3. 2	5. ½	7. 2	9. 5/2	11. ∞	13. -∞
15. 0	17. 1, -1, does not exist		19. 0, does not exist, does not exist			21. 2
23. -1/2	25. 0	27. 2, 4, 2x	29. 1, 0, ∞, -∞	31. Cont.	33. Cont.	
35. -1, 0	37. 1	39. 0	41. Does not exist	45. 2	47. 1	

Even answers:

2. $\frac{2}{3}$ 4. $\frac{5}{7}$ 6. $\frac{1}{2}$ 8.6   10. $\frac{1}{7}$ 12. $-\infty$ 14. $\frac{2}{3}$ 16. $\frac{3}{2}$

18.a. $-\infty$ b. ∞ c. DNE   20.a. 3   b. 2   c. DNE   22. $\frac{1}{4}$ 24. ∞

26. 1   28.a. 3   b.12   c. $3x^2$ 30.a. ∞ b. 0   c. $-\infty$ d. ∞

32. not continuous at x = 0   34. not continuous at x = 1   44. 1   46. $\frac{1}{2}$

B18: Misc. Derivatives

1. a. k = 4 b. k = 6

2. a. A&B b. greater than c. insert graph

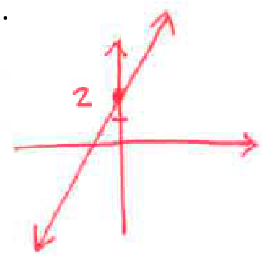
3. a. $f'(2)=\dfrac{1}{2\sqrt{2}}$ b. $f'(3)=-\dfrac{1}{9}$ c. $f'(3)=12$ d) $f'(2)=13$

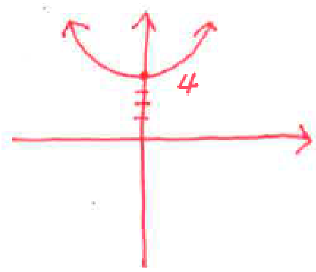
4. *f* is continuous at *x* = 2 (verify this first since if it is not continuous at *x* = 2, then it is not differentiable at *x* = 2. Use the definition of continuous to verify continuity) . Then we look at differentiable:

$f'(x)=\begin{cases}2x & if\ \ x<2\\ 4 & if\ x>2\end{cases}$ and $\lim\limits_{x\to 2^-} f'(x)=\lim\limits_{x\to 2^+} f'(x)=4$ so $\lim\limits_{x\to 2} f'(x)$ = 4. Therefore *f* is differentiable at *x* = 2.

5. a) $(-\infty,-2)U(-2,\infty)$ b) $(-\infty,-3)U(-3,1)U(1,\infty)$ c. $(-\infty,0)U(0,\infty)$

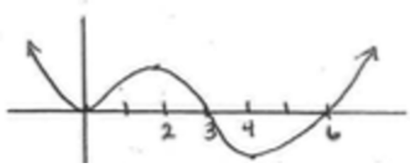
6. a) point (0,2), slope = 3 b) point (0, 4), when x = 0, m = 0; when x < 0, m is negative; when x > 0, m is positive.





B19 – graphs of derivatives review

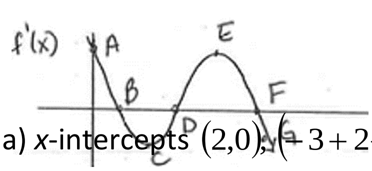
1. Line 1 gives you the *x*-intercepts
Line 2 tells you about the critical points
Lines 3 and 4 tell you about concavity

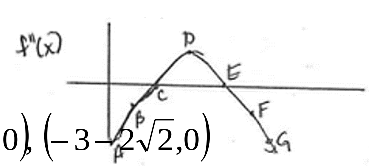


2. a) $(-\infty,-3)\cup(3,\infty)$
b) $f'(x)$ is positive
c) $f'(3)=0$ because local min at *x* = 3
d) possible answer: *x* = 0

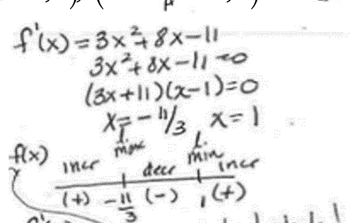
3. a) *f*(*x*) is increasing on (3, 8)
b) $f(x)$ is concave down on (-5, 0). We know nothing about critical points.
c) There is a critical point at *x* = 3. We do NOT know if it is a max, min or point of inflection.
d) $f(x)$ is increasing on $(-\infty,10)\cup(20,\infty)$
 $f(x)$ is decreasing on (10, 20)
 There is a local max of $f(10)$ at *x* = 10
 There is a local min of $f(20)$ at *x* = 20

4.





5. a) *x*-intercepts $(2,0)$, $(-3+2\sqrt{2},0)$, $(-3-2\sqrt{2},0)$
b) *y*-intercept (0, -2)



5c) local max = $\frac{1156}{27}$ at $x = -\frac{11}{3}$

d) local min = -8 at x = 1

e) point of inflection $\left(-\frac{4}{3}, \frac{470}{27}\right)$

f) increasing on $\left(-\infty, -\frac{11}{3}\right) \cup (1, \infty)$

decreasing on $\left(-\frac{11}{3}, 1\right)$

g) concave up on $\left(-\frac{4}{3}, \infty\right)$

concave down on $\left(-\infty, -\frac{4}{3}\right)$

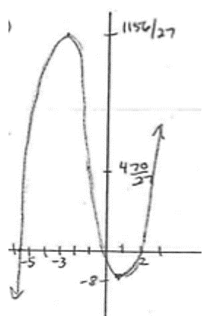
$f''(x) = 6x + 8$

$6x + 8 = 0 \Rightarrow x = -4/3$

f(x) c.d. c.u. e) poi $\left(-\frac{4}{3}, \frac{470}{27}\right)$

$f''(x)$ (−) $-\frac{4}{3}$ (+)

h)



B20

1a) show work 1b) x = 1

2a) show work 2b) x = 1

3) $A(x) = 8x - 2x^2$; 8 units^2

4. a. Since area = length · width, $A = x \cdot$ width; width $= \frac{A}{x}$.

b. perimeter = 2 · length + 2 · width $= 2x + \frac{2A}{x}$

4c) $\sqrt{A}$ for both the length and width

4d) the square with the given area

5) $V = x(8-2x)(15-2x)$; $\frac{5}{3}$ cm

6a) $V(x) = x(1-x)(1-2x); 0 < x < \frac{1}{2}$

6b) $\frac{3-\sqrt{3}}{6} \approx 0.21$

7) (0 , 0), (1 , 1), (-1 , -1); $\frac{\sqrt{3}}{3}$

8) x = 20cm, h = 10cm 9) 400 sets 10a) $d(t)=\sqrt{(60t)^2+(10-80t)^2}, t \ge 0$

10b) 12:48 PM 10c) 60 km 11) 2ft by 2ft by 2ft

12a) show work 12b) 64π cubic units 13) $\frac{32\pi}{3}u^3$ 14) $\frac{4000\pi\sqrt{3}}{9}$ cubic units

15a) \$6.60; \$6.52 15b) 60mi/hr 15c) AMV; for example, a speed of 60 mi/h may be over the local speed limit or it may be an unsafe speed given the truck design. The load being carried, or the road conditions.

B21

1. -20 and 20 3. 2 ft by 2 ft by 1 ft 5. $5\sqrt{5}$

7. base = $a\sqrt{2}$, height = $\frac{a}{\sqrt{2}}$ 9. $\frac{32\pi a^3}{81}$

11. $r=\frac{1}{\sqrt[3]{\pi}} \approx 0.683\ ft\,; h=\frac{1}{\sqrt[3]{\pi}} \approx 0.683\ ft$ 13. 3 inches

17. sidth $\frac{2a}{\sqrt{3}}$, depth $\frac{2a\sqrt{2}}{\sqrt{3}}$ 19. (1, 2) 21. 500

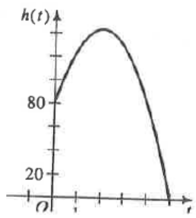
23. $\frac{d^3\sqrt{s_1}}{\sqrt[3]{s_1+\sqrt[3]{s_2}}}$ 24. About 2:23 pm

25. $\frac{50}{7}\ ft \times 25\ ft$

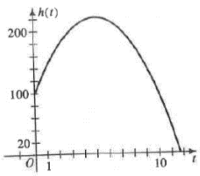
27. there can be no cylindrical part. The tank must be spherical with radius $\sqrt[3]{\frac{75}{\pi}}$

29. a) use approx. 16.71 cm for the rectangle b) use all of the wire for the rectangle

B22



1a) 32 ft/sec   1b) (64-32t) ft/sec ; 32 ft/sec   1c) t = 2 sec
1d) 144 ft   1e) t = 5 sec   1f) $2 < t < 5$ 1g) -32 ft/sec^2
1h) see graph

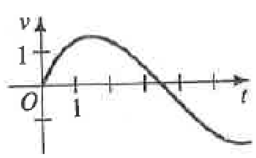


2a) 39.2 m/sec   2b) (49-9.85t) m/sec ; 39.2 m/sec
2c) t = 5 sec   2d) 220.5 m   2e) $t = 5 + 3\sqrt{5}$; 11.7 sec
2f) $5 < t < 5 + 3\sqrt{5}$ 2g) -9.8 m/sec^2 2h) see graph

3a) 2 m   3b) 1 m/sec   3c) $0 < t < 3.5$; $t > 3.5$ 3d) t = 6 sec
3e) 1 m/sec ; 1 m/sec ; 0.5 m/sec   3f) 1.5 sec ; t = 5 sec   3g) 3 m

4a) 1 m   4b) -0.5 m/sec   4c) (0,2); (2, 6)   4d) 2.8 sec
4e) -0.5 m/sec ; 0 m/sec ; 2 m/sec   4f) t = 3 sec ; t = 1/2 sec   4g) 3 m

5.



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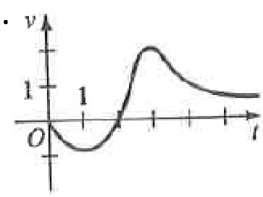

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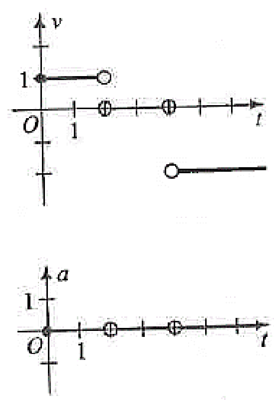

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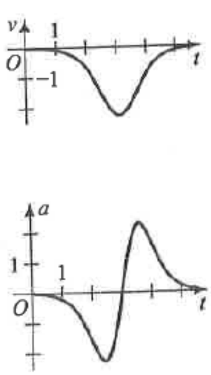
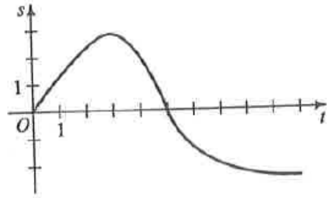


9.



13a) t = 2 sec 13b) t = 0 sec t = 2 sec 13c) t = 1 sec

14a) 7.5 mi/hr eastward 14b) 15 mi west of Rockford; 1 PM

B23

$f_1' = g_7$
$f_2' = g_1$
$f_{31}' = g_9$
$f_4' = g_4$
$f_5' = g_2$
$f_6' = g_6$
$f_7' = g_{11}$
$f_8' = g_{10}$
$f_9' = g_5$
$f_{10}' = g_8$
$f_{11}' = g_{13}$
$f_{12}' = g_3$
$f_{13}' = g_{12}$

B24

B24 Solutions

1. $s(t) = -t^3 + 7t^2 - 14t + 8$; $-(t-1)(t-2)(t-4)$
$v(t) = -3t^2 + 14t - 14$; $v(t) = 0$ @ $t = 3.215$ and $t = 1.4514$
$a(t) = -6t + 14$; $a(t) = 0$ @ $t = 7/3$

a) $\frac{s(2) - s(0)}{2 - 0} = \frac{0 - 8}{2} = -4$ m/sec

b) $v(2) = -12 + 28 - 14 = 2$ m/sec.

c) $a(2) = -12 + 14 = 2$ m/sec^2

d) Rest @ $v(t) = 0$ @ $t = 3.215$ and $t = 1.4514$
↳ B ↳ A
Right: $t \in (1.45, 3.215)$
Left: $[0, 1.45) \cup (3.215, \infty)$

f) $|s(A) - s(0)| + |s(2) - s(A)| \Rightarrow$ ~~| |~~ $|-.63 - 8| + |0 - (-.63)|$

e) $s(2) - s(0) = -8$ meters

f) $8.63 + .63 = 9.26$

g) Not the same ; turn-around point on the interval

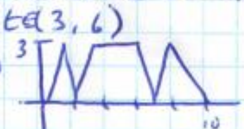
h) v(t) – – + + + + + + + + – –
a(t) 1.45 2.33 3.215
– + – +

Speeding up : $(1.45, 7/3) \cup (3.215, \infty)$
Slowing down $[0, 1.45) \cup (7/3, 3.215)$

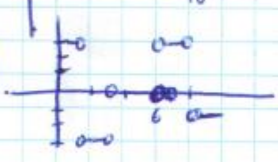
2. Graph is of $v(t)$

a) Left $(2, 7)$; Right $(0, 2) \cup (7, 10)$
@ rest @ $t = 0, 2, 7, 10$

b) $t \in (3, 6)$

c) 

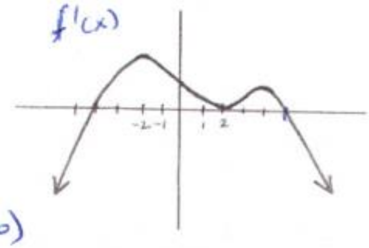
Abs. of $v(t)$ = speed

d) 

e) v(t) + + – – – – – + + +
a(t) 0 + – 2 – 0 0 0 + + – –
+ – + – + – –

Speeding up : $(0, 1) \cup (2, 3) \cup (7, 8)$
Slowing down : $(1, 2) \cup (6, 7) \cup (8, 10)$

B25



1. Given the graph of $y = f'(x)$, the derivative of $y = f(x)$, determine the following about $y = f(x)$:

a) intervals where $y = f(x)$ is increasing/decreasing.

Incr: $(-4, 5)$
dec: $(-\infty, -4) \cup (5, \infty)$

b) x-coordinate of relative maximum/minimum

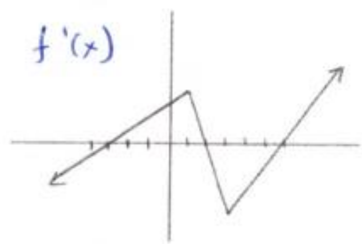
Rel min @ $x = -4$ → $f'(x)$ changes from − to +
Rel Max @ $x = 5$ → $f'(x)$ changes from + to −

c) intervals of concavity

Concave up when slope of $f'(x)$ [$f''(x)$] is +: $(-\infty, -2) \cup (2, 4)$
concave down $(-2, 2) \cup (4, \infty)$

d) x-coordinate of points of inflection

Changes concavity @ $x = -2$
$x = 2$
$x = 4$



2. Given the graph of $y = f'(x)$, the derivative of $y = f(x)$, determine the following about $y = f(x)$:

a) intervals where $y = f(x)$ is increasing/decreasing.

Incr: $(-3, 2) \cup (6, \infty)$
decr: $(-\infty, -3) \cup (2, 6)$

b) x-coordinate of relative maximum/minimum

Rel min @ $x = -3$ and $x = 6$ → $f'(x)$ changes from − to +
Rel Max @ $x = 2$ → $f'(x)$ changes from + to −

c) intervals of concavity

Concave up: $(-\infty, 1) \cup (3, \infty)$
Concave down: $(1, 3)$

d) x-coordinate of points of inflection

$x = 1$
$x = 3$

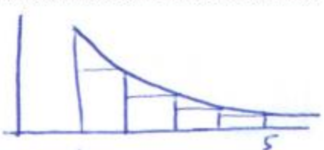
B26

	RRAM	LRAM	MRAM
1.	8.037	5.370	6.593
2.	3.270	3.166	3.220
3.	4.074	2.741	3.185
4.	2.063	2.063	1.875
5.	5.991	5.991	6.519

	RRAM	Reimann Sum
6.	14.5	10
7.	20.074	$21\frac{1}{3}$
8.	19.04	$\frac{46}{3}$
9.	10.5	$\frac{32}{3}$

B27

1. a) Approximate the area under the graph of $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 5$ using the **right endpoints** for four subintervals of equal length. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

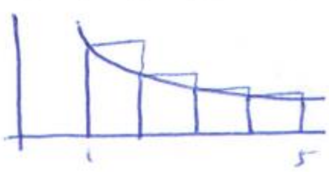


$1\,(f(2)+f(3)+f(4)+f(5))$

$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{77}{60} \approx 1.283$

underestimate

b) Repeat part a) using **left endpoints**.



$1\,(f(1)+f(2)+f(3)+f(4))$

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{25}{12} \approx 2.08\overline{33}$

overestimate

2. Approximate the area under the graph of $f(x) = 25 - x^2$ from $x = 0$ to $x = 5$ using the **midpoints** of five subintervals of equal length. Sketch the graph and the rectangles.

$1\,(f(\tfrac{1}{2})+f(\tfrac{3}{2})+f(\tfrac{5}{2})+f(\tfrac{7}{2})+f(\tfrac{9}{2}))$

$24\tfrac{3}{4} + 22\tfrac{3}{4} + 18\tfrac{3}{4} + 12\tfrac{3}{4} + 4\tfrac{3}{4}$

$80 + 5(\tfrac{3}{4}) = 83\tfrac{3}{4}$ or 83.75

3.

x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

a) Given the table of values above, approximate the area under the curve $y = f(x)$ from $x = -5$ to $x = 5$ using the **left endpoints** for four subintervals.

$2(10) + 3(7) + 1(5) + 4(8)$

$20 + 21 + 5 + 32 = 78$

b) Repeat part a) above using **right endpoints**.

$2(7) + 3(5) + 1(8) + 4(11)$

$14 + 15 + 8 + 44 = 81$

c) Why can you not do this problem with midpoints of four subintervals?

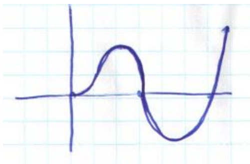
We do not know midpoint f(x) values

d) Approximate the area under the curve using the midpoints of 2 subintervals.

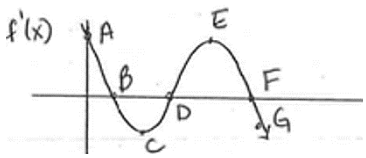
$5(7) + 5(8) =$

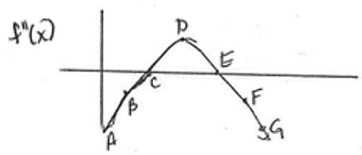
$35 + 40 = 75$

B28 – Applications of Derivative Review (oldR1)

1. 

2a) $(-\infty,-3)\cup(3,\infty)$ 2b) $f'(x)$ is positive 2c) $f'(3)=0$ because local min at $x = 3$
d) $f''(x)=0$ at about $x=0$.

3. 



4a) 89 units/sec 4b) 138 units/sec 4c) 16 units/sec^2 4d) 22 units/sec^2

4e) $\left(-\infty,\frac{5-\sqrt{17}}{4}\right)\cup\left(\frac{5+\sqrt{17}}{4},\infty\right)$

5a) $A(x,h)=2x^2+4xh$ 5b) $h=\frac{1000}{x^2}$ 5c) $A(x)=2x^2+\frac{4000}{x};\ x>0$

5d) Show $A'(x)=0$ when $x = h = 10$.

6. Position: 54 units right of the origin
Velocity : 25 units/sec
Acceleration : 6 units/sec^2

7a) 3 sec 7b) 400 ft 7c) 7 sec 7d) -64 ft/sec